A Bayesian Split Population Survival Model for Duration Data With Misclassified Failure Events

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Contents

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SUPPLEMENTAL APPENDIX

This Supplemental Appendix is divided into six main sections. In the first section, we derive the closed form of the full conditional distributions of $\pi(\Sigma_\beta|\boldsymbol{\beta}_i)$ and $\pi(\Sigma_\gamma|\boldsymbol{\gamma}_i)$ that are required for Gibbs sampling in Step 1 of the MCMC method (with the slice sampling scheme) used for estimating the Bayesian Misclassified Failure (MF) Weibull model. This section also presents the steps for slice sampling for $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, and ρ. The second section provides a detailed derivation of the log-likelihood function of our parametric MF model with time-varying covariates that is stated in equation (9) of the main paper. This is followed by a discussion of the main properties of this parametric MF model, and the description of the MF exponential model and its estimation via MCMC methods. The third section presents and discusses the additional results (including tables and figures) from the Monte Carlo simulation analyses that were mentioned—but not presented in detail to save space—in the main paper. The fourth section describes the steps developed to calculate the Deviance Information Criterion (DIC) from the estimated Bayesian non-MF and MF Weibull models estimated on the Buhaug *et al* sample, and the discussion of the DIC results obtained from these models. The fifth section presents all the additional tables, figures and diagnostic checks generated from the application of our Bayesian MF Weibull model to the Buhaug *et al* (2009) data. The sixth section discusses in detail the Bayesian MF Weibull model results (including all tables and figures) for the Reenock, Bernhard and Sobek (2007) democratic survival application.

I Full Conditional Distributions and Slice Sampling

We first derive the closed form of the full conditional distributions of $\pi(\Sigma_\beta|\boldsymbol{\beta_i})$ and $\pi(\Sigma_\gamma|\boldsymbol{\gamma_i})$ in Step 1 of the MCMC estimation of our MF Wiebull model:

1. Σ_β:

$$\begin{aligned}
\pi(\Sigma_\beta|\boldsymbol{\beta}) &\propto \pi(\boldsymbol{\beta}|\boldsymbol{\mu}_\beta = \mathbf{0}, \Sigma_\beta) \times \pi(\Sigma_\beta) \\
&\propto |\Sigma_\beta|^{-\frac{1}{2}} \exp\Big\{-\frac{1}{2}\Big(\boldsymbol{\beta}'\Sigma_\beta^{-1}\boldsymbol{\beta}\Big)\Big\} \times |\Sigma_\beta|^{-\frac{\nu_1+p_1+1}{2}} \exp\Big\{-\frac{1}{2}\mathrm{tr}\Big(\mathbf{I}_{p_1}\Sigma_\beta^{-1}\Big)\Big\} \\
&= |\Sigma_\beta|^{-\frac{1+\nu_1+p_1+1}{2}} \exp\Big\{-\frac{1}{2}\mathrm{tr}\Big(\boldsymbol{\beta}\boldsymbol{\beta}'\Sigma_\beta^{-1}\Big)\Big\} \times \exp\Big\{-\frac{1}{2}\mathrm{tr}\Big(\mathbf{I}_{p_1}\Sigma_\beta^{-1}\Big)\Big\} \\
&= |\Sigma_\beta|^{-\frac{1+\nu_1+p_1+1}{2}} \exp\Big\{-\frac{1}{2}\mathrm{tr}\Big((\boldsymbol{\beta}\boldsymbol{\beta}' + \mathbf{I}_{p_1})\Sigma_\beta^{-1}\Big)\Big\} \\
&\sim \mathrm{IW}(\boldsymbol{\beta}\boldsymbol{\beta}' + \mathbf{I}_{p_1}, \quad 1+\nu_1)
\end{aligned} \tag{A.1}$$

2. Σ_γ:

$$\begin{aligned}
\pi(\Sigma_\gamma|\boldsymbol{\gamma}) &\propto \pi(\boldsymbol{\gamma}|\boldsymbol{\mu}_\gamma = \mathbf{0}, \Sigma_\gamma) \times \pi(\Sigma_\gamma) \\
&\propto |\Sigma_\gamma|^{-\frac{1}{2}} \exp\Big\{-\frac{1}{2}\Big(\boldsymbol{\gamma}'\Sigma_\gamma^{-1}\boldsymbol{\gamma}\Big)\Big\} \times |\Sigma_\gamma|^{-\frac{\nu_1+p_2+1}{2}} \exp\Big\{-\frac{1}{2}\mathrm{tr}\Big(\mathbf{I}_{p_2}\Sigma_\gamma^{-1}\Big)\Big\} \\
&= |\Sigma_\gamma|^{-\frac{1+\nu_2+p_2+1}{2}} \exp\Big\{-\frac{1}{2}\mathrm{tr}\Big(\boldsymbol{\gamma}\boldsymbol{\gamma}'\Sigma_\gamma^{-1}\Big)\Big\} \times \exp\Big\{-\frac{1}{2}\mathrm{tr}\Big(\mathbf{I}_{p_2}\Sigma_\gamma^{-1}\Big)\Big\} \\
&= |\Sigma_\gamma|^{-\frac{1+\nu_2+p_2+1}{2}} \exp\Big\{-\frac{1}{2}\mathrm{tr}\Big((\boldsymbol{\gamma}\boldsymbol{\gamma}' + \mathbf{I}_{p_2})\Sigma_\gamma^{-1}\Big)\Big\} \\
&\sim \mathrm{IW}(\boldsymbol{\gamma}\boldsymbol{\gamma}' + \mathbf{I}_{p_2}, \quad 1+\nu_2)
\end{aligned} \tag{A.2}$$

We next turn to describe the slice sampling algorithm for estimating $\boldsymbol{\beta}$,$\boldsymbol{\gamma}$ and ρ. Following the current practice in Bayesian mixture survival models, we use univariate slice sampler with stepout and shrinkage (Neal 2003) in Step 2, where the closed form of the full conditional distribution is intractable. We also follow the modifications made in 'BayesMixSurv' R package (Mahani, Mansour, and Mahani 2016). Below are the steps to perform slice sampling for $\boldsymbol{\beta}$ (note that slice sampling for $\boldsymbol{\gamma}$ and ρ is done in the exact same manner and hence not described here to avoid repetition):

For $\boldsymbol{\beta}_p$, $p = \{1, ..., P\}$,

- **Step 0.** Choose an arbitrary starting point β_{p_0} and size of the slice w, and set $i = 0$.

- **Step 1.** Draw y from Uniform$(0, f(\beta_{p_0}))$ defining slice $S = \{\beta_p : y < f(\beta_p)\}$, where

$$\begin{aligned}
f(\beta_p) &\propto \pi(\beta_p|\boldsymbol{\beta}_{-p}, \mathbf{C}, \mathbf{X}, \mathbf{Z}, \mathbf{t}, \mathbf{t0}, \boldsymbol{\gamma}) && \text{if exponential} \\
&\propto \pi(\beta_p|\boldsymbol{\beta}_{-p}, \mathbf{C}, \mathbf{X}, \mathbf{Z}, \mathbf{t}, \mathbf{t0}, \boldsymbol{\gamma}, \rho) && \text{if Weibull.}
\end{aligned} \tag{A.3}$$

- **Step 2.** Find an interval, $I = (L, R)$, around β_{p_0} that contains all, or much, of the slice, where the initial interval is determined as:

$$\begin{aligned} & u \sim \text{Uniform}(0, w) \\ & L = \beta_{p_0} - u \qquad , \\ & R = \beta_{p_0} + (w - u) \end{aligned} \tag{A.4}$$

and expand the interval until its ends are outside the slice or until the limit on steps (limit on steps $= m$) is reached ("stepping-out" procedure), by comparing y and $(f(L), f(R))$:

$$\begin{aligned} & J = \text{Floor}(\text{Uniform}(0, m)) \\ & K = (m - 1) - J \\ & \text{Repeat while } J > 0 \text{ and } y < f(L): \\ & L = L - w, J = J - 1 \\ & \text{Repeat while } K > 0 \text{ and } y < f(R): \\ & R = R + w, K = K - 1 \end{aligned} \tag{A.5}$$

- **Step 3.** Draw a new point β_{p_1} from the part of the slice within this interval I, and shrink the interval on each rejection ("shrinkage" procedure):

$$\begin{aligned} \text{Repeat:} \qquad & \beta_{p1} \sim \text{Uniform}(L, R) \\ & \text{if } y < f(\beta_{p_1}), \text{ accept } \beta_{p_1} \text{ and exit loop} \\ & \text{if } \beta_{p_1} < \beta_{p_0}, \text{ then } L = \beta_{p_1} \\ & \qquad \text{else } R = \beta_{p_1}. \end{aligned} \tag{A.6}$$

- **Step 4.** Set $i = i + 1$, $\beta_{p_0} = \beta_{p_1}$, and go to Step 1.
- **Step 5.** After N iterations, summarize the parameter estimates using all sampled values (via, e.g., credible intervals or posterior means).

II Time-Varying Misclassified Failure Model

Log-Likelihood and Properties

The likelihood function of the general parametric MF model that is developed and defined from equations (1)-(5) in the paper is given by:

$$L = \prod_{i=1}^{N} [\alpha_i + (1 - \alpha_i) f(t_i | \mathbf{X}_i, \boldsymbol{\beta})]^{C_i} [(1 - \alpha_i) S(t_i | \mathbf{X}_i \boldsymbol{\beta})]^{1 - C_i}, \tag{A.7}$$

and the model's log-likelihood is

$$\ln L = \sum_{i=1}^{N} \{\widetilde{C}_i \ln[\alpha_i + (1-\alpha_i) f(t_i|\mathbf{X}_i, \boldsymbol{\beta})] + (1-\widetilde{C}_i) \ln[(1-\alpha_i) S(t_i|\mathbf{X}_i, \boldsymbol{\beta})]\}. \tag{A.8}$$

Further, recall from the text that the log-likelihood function of the standard general parametric survival model with time-varying covariates is

$$\ln L = \sum_{i=1}^{N} \left\{ \tilde{C}_{ij} \ln \left[\frac{f(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})}{S(t0_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})} \right] + (1-\tilde{C}_{ij}) \ln \left[\frac{S(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})}{S(t0_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})} \right] \right\}, \tag{A.9}$$

where $\widetilde{C}_{ij} = 0$ denotes all censored observations that are correctly recored, while $\widetilde{C}_{ij} = 1$ are non-censored (i.e., "failed") observations, which may be contaminated with cases that are actually censored cases (i.e., $C_{ij} = 0$). Hence, given $\widetilde{C}_{ij} = 0$ and $\widetilde{C}_{ij} = 1$, we can define the probability of misclassification as:

$$\alpha_{ij} = \Pr(C_{ij} = 0|\widetilde{C}_{ij} = 1). \tag{A.10}$$

Incorporating α_{ij}, the unconditional density of an event happening is

$$Pr(\alpha_{ij} = 1) + Pr(\alpha_{ij} = 0) \Pr(t_{ij} \leq T_{ij}) = \alpha_{ij} + (1-\alpha_{ij}) \frac{f(t_{ij})}{S(t0_{ij})}, \tag{A.11}$$

with a corresponding unconditional survival function of

$$Pr(\alpha_{ij} = 0) \Pr(t_{ij} > T_{ij}) = (1-\alpha_{ij}) \frac{S(t_{ij})}{S(t0_{ij})}. \tag{A.12}$$

Combining these two parts and using equation A.9, the log-likelihood function of the parametric MF model with time-varying covariates is defined as:

$$\ln L = \sum_{i=1}^{N} \left\{ \widetilde{C}_{ij} \ln \left[\alpha_{ij} + (1-\alpha_{ij}) \frac{f(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})}{S(t0_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})} \right] + (1-\widetilde{C}_{ij}) \ln \left[(1-\alpha_{ij}) \frac{S(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})}{S(t0_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})} \right] \right\}, \tag{A.13}$$

where $\alpha_{ij} = \dfrac{\exp(\mathbf{Z}_{ij}\boldsymbol{\gamma})}{1+\exp(\mathbf{Z}_{ij}\boldsymbol{\gamma})}$.

Building on the preceding discussion, note that if one were to define a probability of non-misclassification as $\delta_{ij} = 1 - \alpha_{ij}$ and substitute this quantity into Equation A.11, the log-likelihood would be defined as:

$$\ln L = \sum_{i=1}^{N} \left\{ C_{ij} \ln \left[(1-\delta_{ij}) + \delta_{ij} \frac{f(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})}{S(t0_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})} \right] + (1-C_{ij}) \ln \left[\delta_{ij} \frac{S(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})}{S(t0_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})} \right] \right\}, \tag{A.14}$$

which is symmetric to the log likelihood of the split-population survival model:

$$\ln L = \sum_{i=1}^{N} \left\{ \tilde{C}_{ij} \ln \left[\delta_{ij} \frac{f(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})}{S(t0_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})} \right] + (1 - \tilde{C}_{ij}) \ln \left[(1 - \delta_i) + \delta_{ij} \frac{S(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})}{S(t0_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta})} \right] \right\}. \quad \text{(A.15)}$$

This implies that some properties of the cure model also hold for the MF model, including (i) the reduction of the latter to a normal parametric model whenever $\delta = 1$ or $\alpha = 0$ and (ii) parameter identification even in the case where identical covariates are included in $\mathbf{Z}$ and $\mathbf{X}$.[1]

Misclassified Failure Exponential Model

To develop the MF exponential model, we require to first define the density function and survival function in this case which are respectively:

$$\begin{aligned} f(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta}) &= \exp(\mathbf{X}_{ij}\boldsymbol{\beta})\exp(-\exp(\mathbf{X}_{ij}\boldsymbol{\beta})t_{ij}) \\ S(t_{ij}|\mathbf{X}_{ij}, \boldsymbol{\beta}) &= \exp(-\exp(\mathbf{X}_{ij}\boldsymbol{\beta})t_{ij}). \end{aligned} \quad \text{(A.16)}$$

Following the steps taken to define the log-likelihood function in equation (9) in the main paper, the log-likelihood function of the MF exponential model with time varying covariates is:

$$\begin{aligned} &\ln L(\beta, \gamma) \\ &= \sum_{i=1}^{N} \left\{ \widetilde{C}_{ij} \ln \left[\alpha_{ij} + (1 - \alpha_{ij}) \frac{\exp(\mathbf{X}_{ij}\boldsymbol{\beta})\exp(-\exp(\mathbf{X}_{ij}\boldsymbol{\beta})t_{ij})}{\exp(-\exp(\mathbf{X}_{ij}\boldsymbol{\beta})t0_{ij})} \right] \right. \\ &\qquad \left. + (1 - \widetilde{C}_{ij}) \ln \left[(1 - \alpha_{ij}) \frac{\exp(-\exp(\mathbf{X}_{ij}\boldsymbol{\beta})t_{ij})}{\exp(-\exp(\mathbf{X}_{ij}\boldsymbol{\beta})t0_{ij})} \right] \right\}, \end{aligned} \quad \text{(A.17)}$$

where $\mathbf{X}_{ij}$ is the i^{th} row of the covariate matrix $\mathbf{X}$ at time j and $\alpha_{ij} = \frac{\exp(\mathbf{Z}_{ij}\boldsymbol{\gamma})}{1 + \exp(\mathbf{Z}_{ij}\boldsymbol{\gamma})}$. As shown in (A.17), the time-varying MF exponential model accounts for the probability of misclassified failure while estimating the effect of the covariates that influence the survival of the event of interest (assumed to be exponentially distributed).

While the MF exponential model with time-varying covariates can be estimated by maximum likelihood using, for example, BFGS,[2] we estimate this model via the MCMC algorithm employed for Bayesian inference. We thus label our model as Bayesian MF exponential model

[1] See Box-Steffensmeier and Zorn (1999, 5) for a discussion of these properties in the context of the split-population model.

[2] The Broyden, Fletcher, Goldfarb, Shannon (BFGS) method in the R *optim* function. In our Monte Carlo analysis, we briefly assess the properties of the MF exponential model estimated by BFGS.

given the use of MCMC estimation. To conduct Bayesian inference, we need to assign a prior for each of the MF exponential model's two parameters – $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ – and then define the conditional posterior distribution of these parameters. Following standard practice, we assign the multivariate normal prior to $\boldsymbol{\beta} = \{\beta_1, ..., \beta_{p_1}\}$ and $\boldsymbol{\gamma} = \{\gamma_1, ..., \gamma_{p_2}\}$:

$$\boldsymbol{\beta} \sim \mathrm{MVN}_{p_1}(\mu_\beta, \Sigma_\beta), \qquad \boldsymbol{\gamma} \sim \mathrm{MVN}_{p_2}(\mu_\gamma, \Sigma_\gamma)$$

$$\Sigma_\beta \sim \mathrm{IW}(S_\beta, \nu_\beta), \qquad \Sigma_\gamma \sim \mathrm{IW}(S_\gamma, \nu_\gamma). \tag{A.18}$$

where we fix $\mu_\beta = \mathbf{0}$ and $\mu_\gamma = \mathbf{0}$ and S_β, ν_β, S_γ, ν_γ are the hyper parameters. Note that we use hierarchical Bayesian modeling to estimate Σ_β and Σ_γ using the Inverse-Wishart (IW) distribution. Given these prior specifications and the hyperparameters, the conditional posterior distributions for $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ parameters in the Bayesian MF exponential model (with time-varying covariates) are:

$$P(\boldsymbol{\beta}|\mathbf{C}, \mathbf{X}, \mathbf{Z}, \mathbf{t}, \mathbf{t0}, \boldsymbol{\gamma}) \propto P(\boldsymbol{\beta}|\mathbf{C}, \mathbf{X}, \mathbf{Z}, \mathbf{t}, \mathbf{t0}, \boldsymbol{\gamma}) \times P(\boldsymbol{\beta}|\boldsymbol{\mu}_\beta, \Sigma_\beta)$$

$$P(\boldsymbol{\gamma}|\mathbf{C}, \mathbf{X}, \mathbf{Z}, \mathbf{t}, \mathbf{t0}, \boldsymbol{\beta}) \propto P(\boldsymbol{\gamma}|\mathbf{C}, \mathbf{X}, \mathbf{Z}, \mathbf{t}, \mathbf{t0}, \boldsymbol{\beta}) \times P(\boldsymbol{\gamma}|\boldsymbol{\mu}_\gamma, \Sigma_\gamma). \tag{A.19}$$

For the sampling scheme, since closed forms for the posterior distributions of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are not available, we use the same MCMC methods with the slice sampling algorithm described in the main paper for the Bayesian MF Weibull Model. The only difference is that, unlike the Weibull model, we ignore the slice sampling for ρ in the Bayesian MF exponential model. The closed form of the full conditional distributions of $P(\Sigma_\beta|\boldsymbol{\beta}_i)$ and $P(\Sigma_\gamma|\boldsymbol{\gamma}_i)$ as well as the slice sampling scheme for $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ is derived and described above (and hence not repeated here).

III Monte Carlo Simulation Figures

This Supplemental Appendix provides a complete presentation of the Monte Carlo (MC) results that are referenced in the main paper. Recall that we conduct 11 MC experiments in total. Experiments 1 and 2 are primarily presented and discussed within the main paper, and evaluate the relative performance of a Bayesian Weibull model and a Bayesian MF Weibull model when one's true data generating process (d.g.p.) is (i) standard Weibull (Experiment 1) or (ii) MF Weibull with a 5% misclassified failure rate (Experiment 2). Experiments 3-4 instead assess the performance of maximum likelihood (specifically, BFGS) estimated versions of the Weibull and MF-Weibull models for these same non-MF Weibull (Experiment 3) and MF-Weibull (Experiment 4) outcome variables. Experiments 5-8 consider an exponentially distributed outcome variable (Experiments 5 and 7), or a MF exponential outcome variable (Experiments 6 and 8), and evaluate the performance of either (i) *Bayesian* Weibull, MF-exponential, and MF Weibull models (Experiments 5-6) or (ii) *BFGS* exponential, Weibull, MF exponential and MF Weibull models (Experiments 7-8). Experiments 9, 10, and 11 revisit the Bayesian Weibull and Bayesian MF Weibull comparisons that we conduct in Experiment 2 under conditions where one's misclassified failure rate is increased from 5% to 8%, 12%, and 15%, respectively. For Experiments 1-11, we compare each relevant model under conditions of $N = 1,000$, $N = 1,500$, and $N = 2,000$. Below, we first present the plotted $\hat{\beta}$ and $\hat{\gamma}$ values for Experiments 1-2 (Figures A.1-A.2) which are referenced in the main text. We then provide a more in-depth interpretation of Experiments 3-11, which includes our reporting of each experiment's corresponding (averaged) parameter estimates, (MC)SE's, and RMSEs.

Experiment 3 compares the performance of (i) a BFGS Weibull model and (ii) a BFGS MF-Weibull model when the true outcome variable's d.g.p. is Weibull and the resultant survival outcome variable contains no instances of misclassified failures. We report this experiment's survival stage MC results in the top portion of Table A.1, and in Figure A.3. The results obtained in Experiment 3 are comparable to those obtained in Experiment 1. In circumstances where a researcher encounters a non-MF Weibull-distributed outcome variable, the BFGS MF Weibull and BFGS non-MF Weibull estimators each perform comparably with respect to bias and efficiency. For instance, one can note in Figure A.3 that each of the MF and non-MF $\hat{\beta}$ distributions virtually identical for each N evaluated. Likewise, the BFGS Weibull and BFGS MF Weibull models' SEs and RMSEs (reported in the top half of Table A.1) are comparable to the third decimal place for each parameter, and N, of interest. Hence, in the case of our BFGS estimators, there does not appear to be a substantial risk in (mis)applying a MF Weibull model

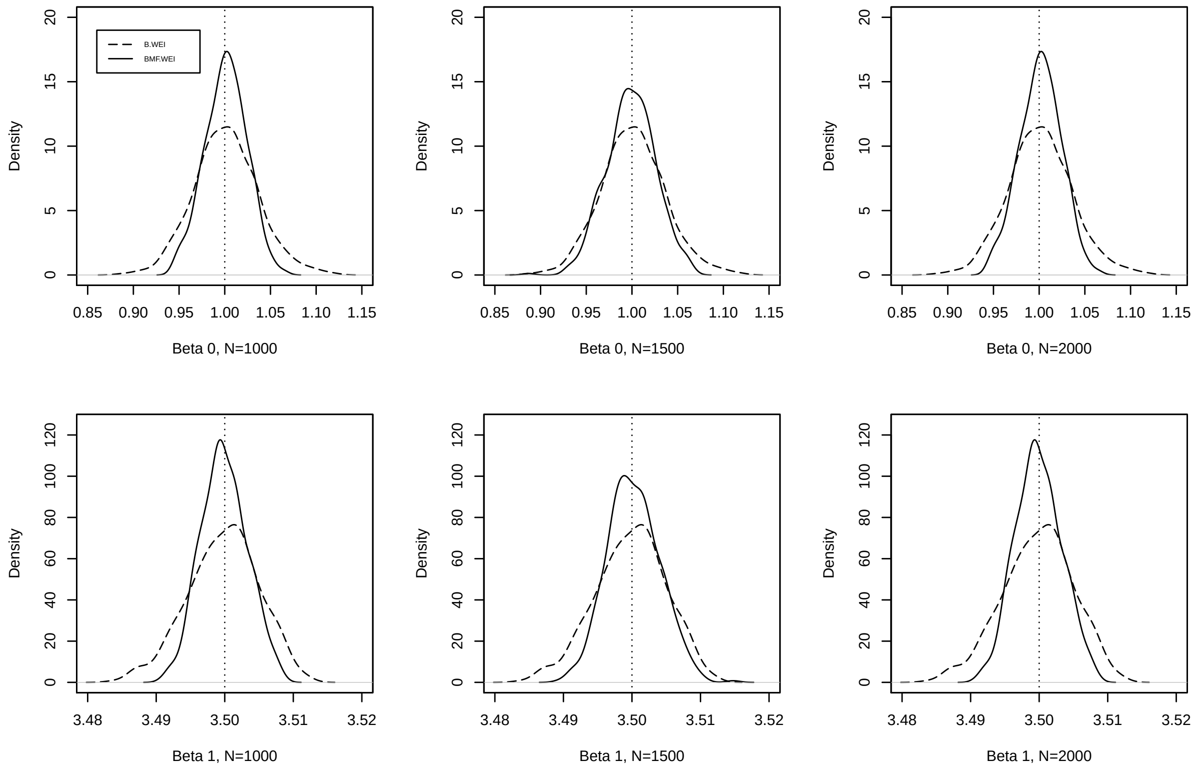


Figure A.1: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 1

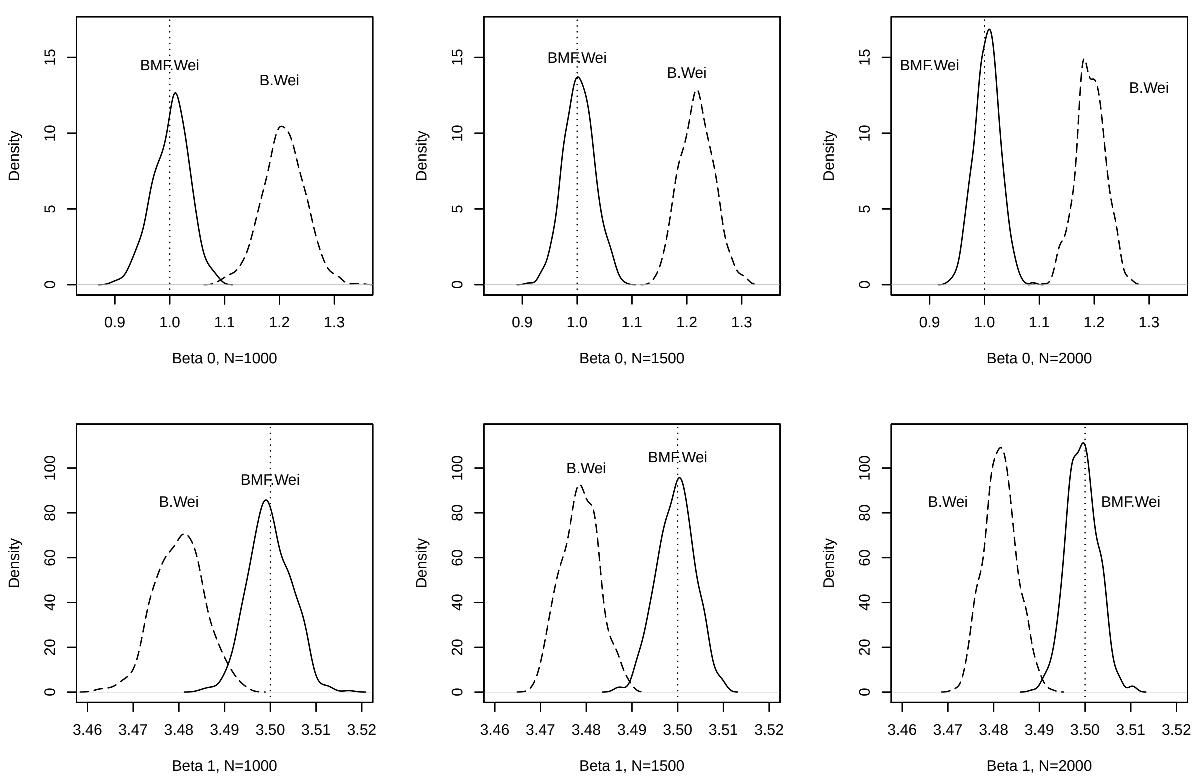


Figure A.2: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 2

to a non-MF Weibull-distributed outcome variable.

Table A.1: Maximum Likelihood β-Estimates for Experiments 3 and 4

	Experiment 3: Non-MF Weibull D.G.P.						
#Obs.	Model	$\hat{\beta}_0$	$SE(\hat{\beta}_0)$	$RMSE(\hat{\beta}_0)$	$\hat{\beta}_1$	$SE(\hat{\beta}_1)$	$RMSE(\hat{\beta}_1)$
1,000	BFGS Weibull	1.003	0.033	0.025	3.500	0.005	0.004
	BFGS MF Weibull	1.003	0.033	0.025	3.500	0.005	0.004
1,500	BFGS Weibull	1.001	0.027	0.022	3.500	0.004	0.003
	BFGS MF Weibull	1.001	0.027	0.022	3.500	0.004	0.003
2,000	BFGS Weibull	0.999	0.023	0.020	3.500	0.003	0.003
	BFGS MF Weibull	0.999	0.023	0.020	3.500	0.003	0.003
	Experiment 4: MF Weibull D.G.P.						
#Obs.	Model	$\hat{\beta}_0$	$SE(\hat{\beta}_0)$	$RMSE(\hat{\beta}_0)$	$\hat{\beta}_1$	$SE(\hat{\beta}_1)$	$RMSE(\hat{\beta}_1)$
1,000	BFGS Weibull	1.212	0.042	0.212	3.480	0.006	0.020
	BFGS MF Weibull	1.005	0.033	0.026	3.499	0.005	0.004
1,500	BFGS Weibull	1.217	0.035	0.217	3.479	0.005	0.021
	BFGS MF Weibull	1.002	0.027	0.022	3.500	0.004	0.003
2,000	BFGS Weibull	1.193	0.029	0.193	3.481	0.004	0.019
	BFGS MF Weibull	1.002	0.024	0.019	3.500	0.003	0.003

Note: True parameter values are $\beta_0 = 1$ and $\beta_1 = 3.5$.

Table A.2: Maximum Likelihood γ-Estimates for Experiment 4

	Experiment 4: MF Weibull D.G.P.									
#Obs.	Model	$\hat{\gamma}_0$	$SE(\hat{\gamma}_0)$	$RMSE(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$SE(\hat{\gamma}_1)$	$RMSE(\hat{\gamma}_1)$	$\hat{\gamma}_2$	$SE(\hat{\gamma}_2)$	$RMSE(\hat{\gamma}_2)$
1,000	BFGS MF Weibull	-1.543	1.148	0.974	2.079	0.436	0.366	3.401	0.625	0.581
1,500	BFGS MF Weibull	-1.362	0.936	0.896	2.041	0.353	0.293	3.425	0.522	0.512
2,000	BFGS MF Weibull	-1.455	0.877	0.801	2.061	0.332	0.268	3.422	0.467	0.496

Note: True parameter values are $\gamma_0 = -2$, $\gamma_1 = 2$, and $\gamma_2 = 3$.

On the other hand, MC Experiment 4 suggests that there *is* a non-negligible risk in (mis)applying a standard BGFS Weibull model to a MF Weibull-distributed outcome variable. Specifically, we observe in Figure A.4, and in the bottom half of Table A.1, that the BFGS Weibull model's $\hat{\beta}$'s generally overestimate β_0 and underestimate β_1, relative to the BFGS MF Weibull estimator. As a result, the corresponding RMSEs reported for Experiment 4 in Table A.1 consistently favor the BFGS MF Weibull model over the BFGS Weibull model by a factor of five to ten. The BFGS Weibull model's averaged SEs in Table A.1 are each also noticeably larger than those of the BFGS MF Weibull, no matter the β parameter evaluated, or the number of observations considered. Together this suggests that the BFGS MF Weibull model—as was the case for the Bayesian MF Weibull model – is preferable to the standard Weibull esti-

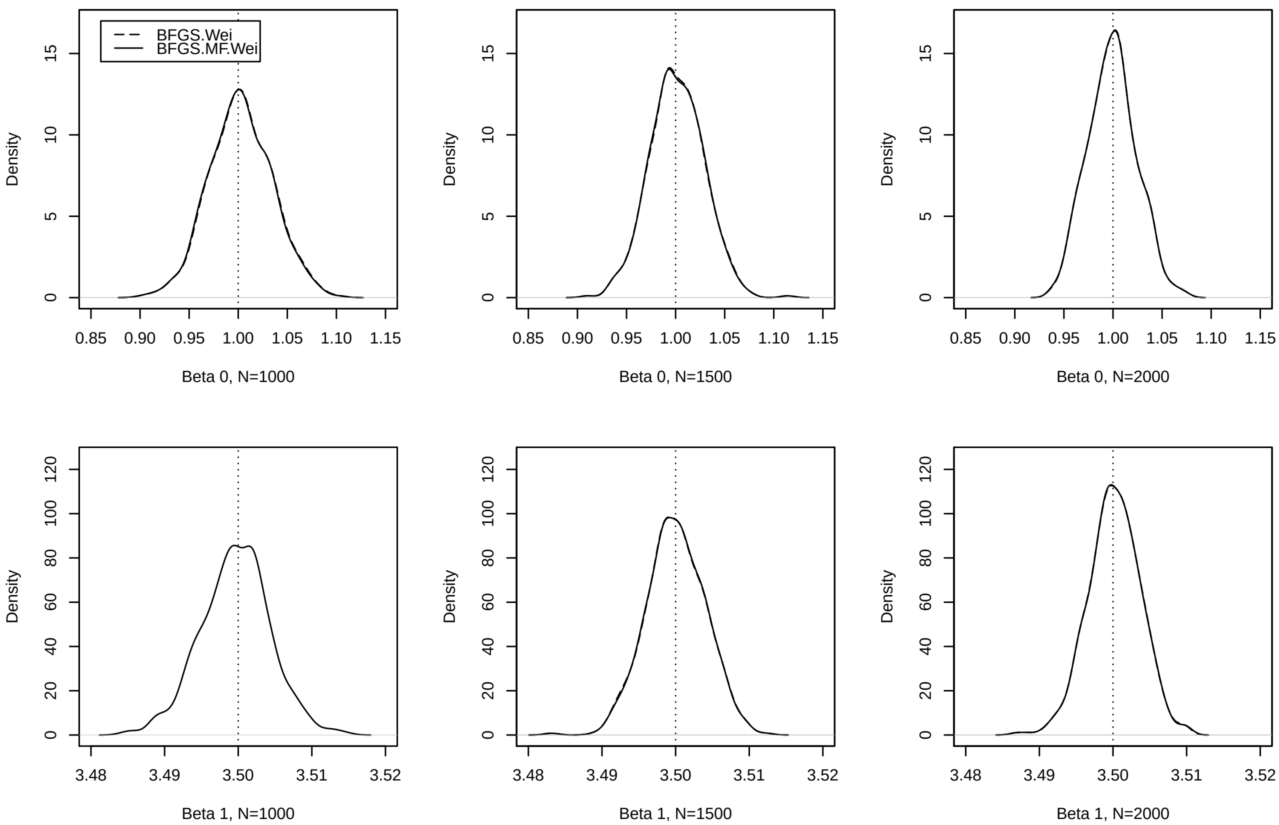


Figure A.3: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 3

mator when misclassified failure cases exist within one's Weibull distributed outcome variable. Turning to Table A.2, we can also briefly note here that the BFGS MF Weibull model's $\hat{\gamma}$'s generally exhibit higher bias, and lower efficiency, than either the BFGS MF Weibull $\hat{\beta}$ results in Table A.1, or the Bayesian MF Weibull $\hat{\gamma}$ results obtained in Experiment 2. The latter finding lends support to our main paper's contention that the Bayesian MF Weibull model is likely preferable to the BFGS MF Weibull model for applied research.

We next turn to MC Experiments 5–6, which compare the performance of the Bayesian (MF) Weibull models to Bayesian MF exponential models when the true d.g.p. is either (i) exponential (Experiment 5) or (ii) MF exponential (Experiment 6). We report the results from these two additional MC experiments in Tables A.3-A.4 and in Figures A.5-A.6. Beginning first with Experiment 5, we find here that the Bayesian MF estimators again perform comparably to our non-MF Bayesian estimators when the true d.g.p. contains no misclassified failure cases.

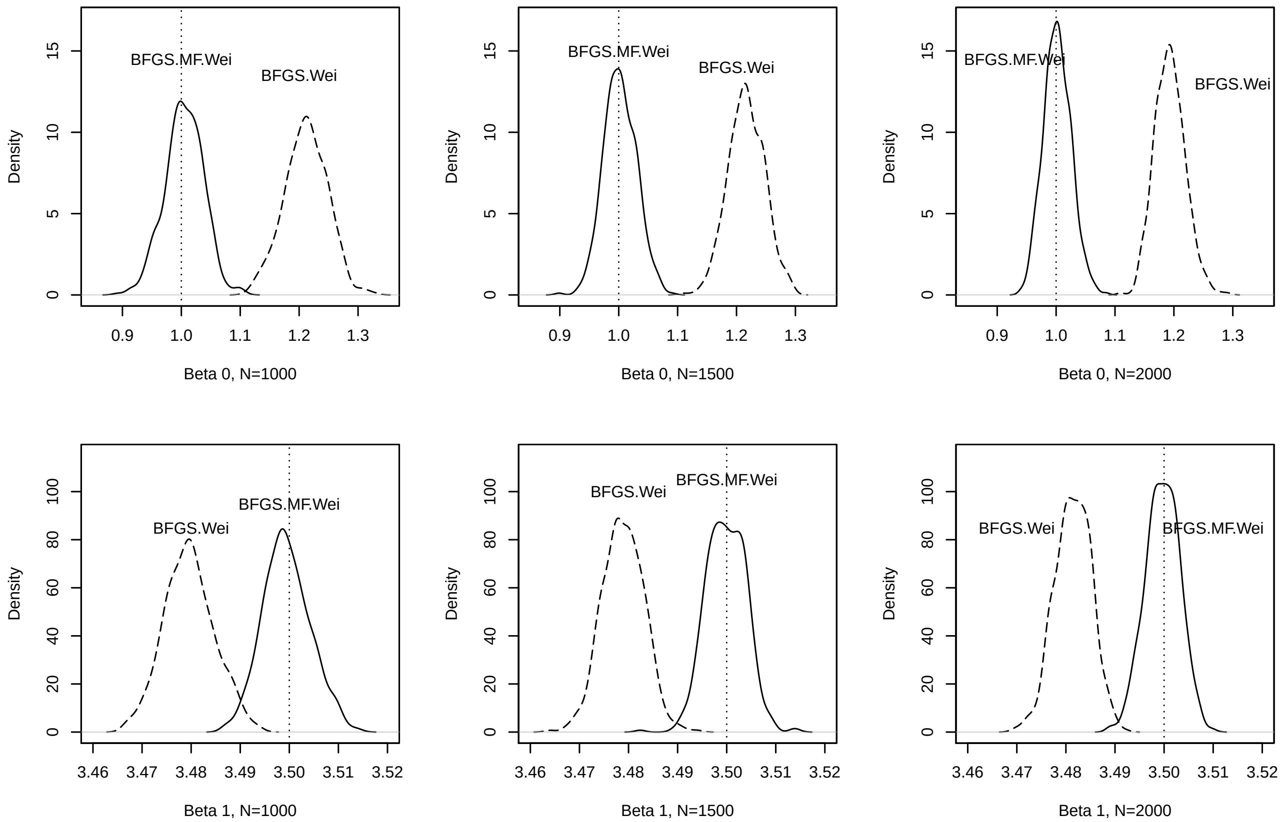


Figure A.4: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 4

For example, we find in Table A.3 that there are many case where we obtain slightly lower RMSEs (and hence less bias) within our Bayesian MF exponential and Bayesian MF Weibull model parameter estimates than in the case of the Bayesian Weibull model. The corresponding averaged $\hat{\beta}$ values reported in Table A.3, and the plots of each $\hat{\beta}$ in Figure A.5, strongly support these conclusions. Nevertheless, and as was the case in Experiments 1 and 2, we do find that the non-MF Bayesian Weibull model exhibits consistently lower MCSEs than either the Bayesian MF exponential or the Bayesian MF Weibull within the top half of Table A.3. This suggests that when no misclassified failure cases exist, non-MF Bayesian survival models are preferable to MF Bayesian survival models for reasons of efficiency and parsimony.

With regards to MC Experiment 6, Tables A.3-A.4 and Figure A.6 reaffirm the conclusions obtained in Experiment 2. For example, the $\hat{\beta}$'s and RMSEs for the Bayesian MF exponential and Bayesian MF Weibull models in Table A.3 consistently indicate that these Bayesian

Table A.3: Markov Chain Monte Carlo (MCMC) β-Estimates for Experiments 5 and 6

	Experiment 5: Non-MF Exponential D.G.P.						
#Obs.	Model	$\hat{\beta}_0$	*MCSE*($\hat{\beta}_0$)	*RMSE*($\hat{\beta}_0$)	$\hat{\beta}_1$	*MCSE*($\hat{\beta}_1$)	*RMSE*($\hat{\beta}_1$)
1,000	Bayes Weibull	1.006	0.0004	0.054	3.499	5.58E-05	0.008
	Bayes MF Exponential	0.999	0.0027	0.053	3.500	0.00036	0.008
	Bayes MF Weibull	0.999	0.0027	0.053	3.500	0.00036	0.008
1,500	Bayes Weibull	1.001	0.0003	0.043	3.500	0.00004	0.006
	Bayes MF Exponential	1.001	0.0018	0.041	3.500	0.00024	0.006
	Bayes MF Weibull	1.001	0.0018	0.041	3.500	0.00024	0.006
2,000	Bayes Weibull	1.004	0.0003	0.045	3.500	0.00004	0.006
	Bayes MF Exponential	0.998	0.0013	0.037	3.500	0.00018	0.005
	Bayes MF Weibull	0.998	0.0013	0.038	3.500	0.00017	0.005
	Experiment 6: MF Exponential D.G.P.						
#Obs.	Model	$\hat{\beta}_0$	*MCSE*($\hat{\beta}_0$)	*RMSE*($\hat{\beta}_0$)	$\hat{\beta}_1$	*MCSE*($\hat{\beta}_1$)	*RMSE*($\hat{\beta}_1$)
1,000	Bayes Weibull	1.337	3.37E-01	0.337	3.464	6.62E-05	0.036
	Bayes MF Exponential	0.997	0.012	0.063	3.500	0.001	0.008
	Bayes MF Weibull	1.007	0.001	0.055	3.499	9.84E-05	0.007
1,500	Bayes Weibull	1.351		0.351	3.461	4.51E-05	0.039
	Bayes MF Exponential	1.004	0.002	0.049	3.499	0.00026	0.007
	Bayes MF Weibull	1.006	3.89E-04	0.048	3.499	4.96E-05	0.007
2,000	Bayes Weibull	1.366	2.43E-04	0.366	3.459	3.51E-05	0.041
	Bayes MF Exponential	1.006	0.003	0.039	3.499	0.00034	0.006
	Bayes MF Weibull	1.008	0.001	0.037	3.499	0.00013	0.006

Note: True parameter values are $\beta_0 = 1$ and $\beta_1 = 3.5$.

Table A.4: Markov Chain Monte Carlo (MCMC) γ-Estimates for Experiment 6

	Experiment 6: MF Exponential D.G.P.									
#Obs.	Model	$\hat{\gamma}_0$	*MCSE*($\hat{\gamma}_0$)	*RMSE*($\hat{\gamma}_0$)	$\hat{\gamma}_1$	*MCSE*($\hat{\gamma}_1$)	*RMSE*($\hat{\gamma}_1$)	$\hat{\gamma}_2$	*MCSE*($\hat{\gamma}_2$)	*RMSE*($\hat{\gamma}_2$)
1,000	Bayes MF Exponential	-1.738	0.134	0.483	1.911	0.103	0.390	3.163	0.139	0.522
	Bayes MF Weibull	-1.672	0.149	0.485	1.952	0.126	0.409	3.225	0.175	0.556
1,500	Bayes MF Exponential	-1.618	0.101	0.436	1.923	0.072	0.329	3.262	0.100	0.486
	Bayes MF Weibull	-1.619	0.088	0.424	1.915	0.061	0.311	3.244	0.080	0.465
2,000	Bayes MF Exponential	-1.742	0.295	0.558	2.056	0.240	0.260	3.296	0.349	0.650
	Bayes MF Weibull	-1.726	0.324	0.580	2.104	0.265	0.299	3.250	0.379	0.601

Note: True parameter values are $\gamma_0 = -2$, $\gamma_1 = 2$, and $\gamma_2 = 3$.

MF survival models exhibit little to no bias in recovered survival-stage parameter estimates when misclassified failure rates exist, whereas the non-MF Bayesian Weibull model consistently overestimates $\hat{\beta}_0$ and underestimates $\hat{\beta}_1$. We find in this case that the corresponding Bayesian MF survival model RMSEs are generally 7-9 times smaller than those of the Bayesian Weibull estimator in Experiment 6, whereas each model's MCSEs are fairly similar across each N evaluated. Taken together, these findings reaffirm Experiment 2's conclusion that the Bayesian MF survival models are preferable to the Bayesian Weibull when one's d.g.p. contains misclassified failure cases. In these regards, we can further note in Table A.4 and Figure A.6 that we cannot draw similar conclusions with regards to the preferability of the Bayesian MF exponential over the Bayesian MF Weibull (or vice-versa), as neither model consistently exhibits superior RMSEs (or MCSEs) over the other in these cases. Given that the MF Weibull nests the MF exponential, but more flexibly handles circumstances where one's hazard rate is non-constant, the Bayesian MF Weibull should typically be favored over the Bayesian MF exponential in applied research.

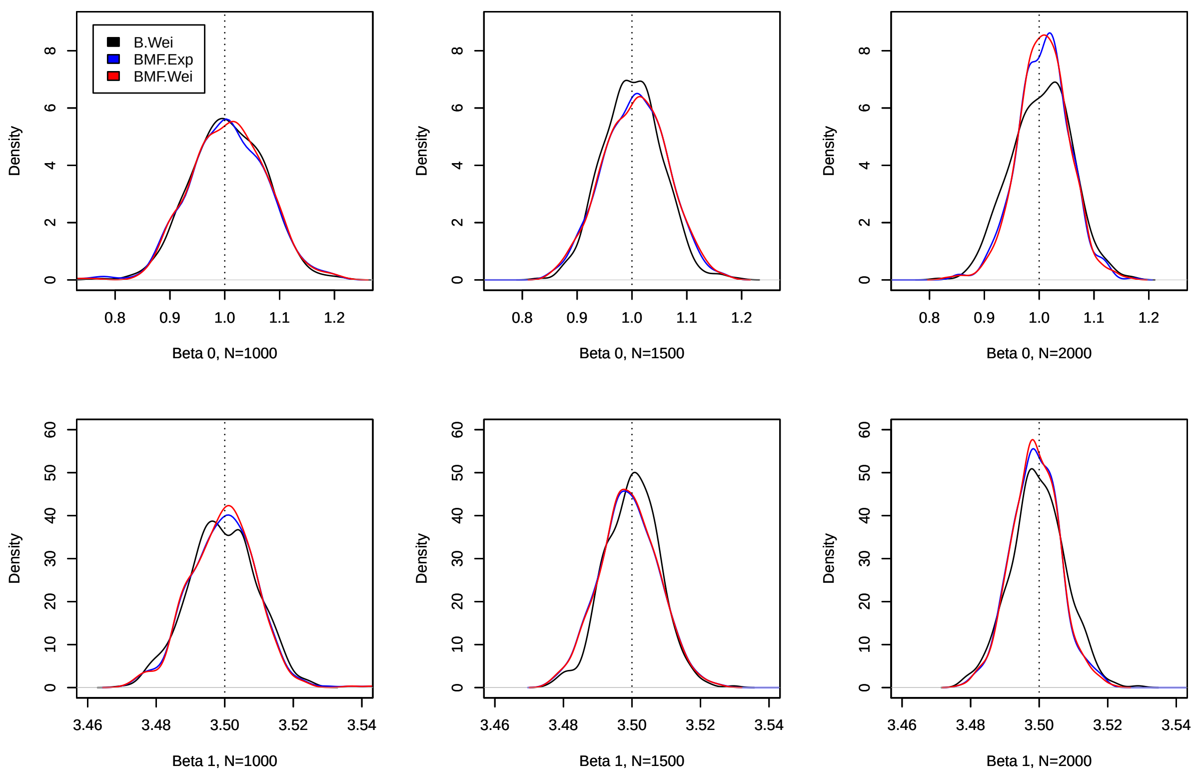


Figure A.5: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 5

We next discuss Experiments 7-8, which evaluate the performance of the BFGS (MF) Weibull and exponential models when the true d.g.p. corresponds to either (i) a simple exponential survival process with no instances of misclassified failures (Experiment 7) or an exponential process with 5% misclassified failure cases (Experiment 8). We report our $\hat{\beta}$ results for these two MC experiments within a Table A.5, and also plot the full distributions of these β parameter estimates within Figures A.7-A.8. In cases where a researcher encounters a non-MF

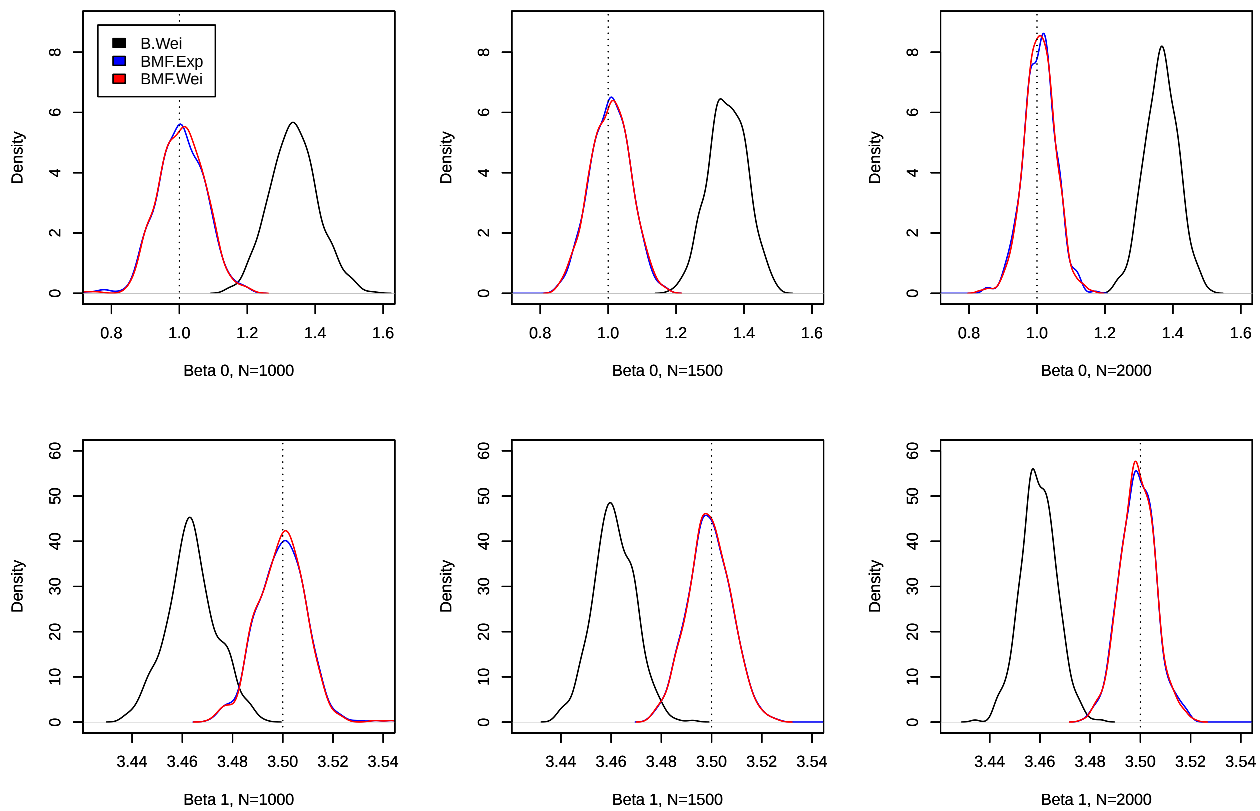


Figure A.6: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 6

exponential-distributed outcome variable, we find in the top-half of Table A.5 (and in Figure A.7) that our BFGS MF survival models perform commensurately with respect to both efficiency and accuracy. For both β parameters, and across each N evaluated, the BFGS MF exponential and BFGS MF Weibull models' averaged parameter estimates are virtually identical (to the third decimal place) to those of the standard exponential and Weibull models. These similarities between the MF and non-MF survival models evaluated within Experiment 7 are also reflected in the SEs and RMSEs reported in the top half of Table A.5, which are generally identical (to the third decimal place) for each BFGS model pair considered.

We can also note in Table A.5 that in cases where one's true d.g.p. is exponential, the BFGS MF Weibull and BFGS MF exponential models recover very similar averaged $\hat{\beta}$ values, and exhibit near-identical RMSEs and SEs. These latter findings generally hold true under Experiment 8 as well, when the true d.g.p. is MF exponential. For instance, each of the $\hat{\beta}$

Table A.5: Maximum Likelihood β-Estimates for Experiments 7 and 8

#Obs.	Model	$\hat{\beta}_0$	$SE(\hat{\beta}_0)$	$RMSE(\hat{\beta}_0)$	$\hat{\beta}_1$	$SE(\hat{\beta}_1)$	$RMSE(\hat{\beta}_1)$
	Experiment 7: Non-MF Exponential D.G.P.						
1,000	BFGS Exponential	1.002	0.066	0.053	3.500	0.010	0.008
	BFGS Weibull	1.001	0.066	0.053	3.500	0.010	0.008
	BFGS MF Exponential	1.001	0.066	0.053	3.500	0.010	0.008
	BFGS MF Weibull	1.000	0.066	0.053	3.500	0.00957	0.008
1,500	BFGS Exponential	1.002	0.053	0.041	3.500	0.008	0.006
	BFGS Weibull	1.002	0.053	0.041	3.500	0.008	0.006
	BFGS MF Exponential	1.002	0.053	0.041	3.500	0.008	0.006
	BFGS MF Weibull	1.002	0.053	0.041	3.500	0.008	0.006
2,000	BFGS Exponential	0.999	0.046	0.037	3.500	0.007	0.005
	BFGS Weibull	0.998	0.046	0.038	3.500	0.007	0.005
	BFGS MF Exponential	0.999	0.046	0.037	3.500	0.007	0.005
	BFGS MF Weibull	0.998	0.046	0.038	3.500	0.007	0.005
	Experiment 8: MF Exponential D.G.P.						
#Obs.	Model	$\hat{\beta}_0$	$SE(\hat{\beta}_0)$	$RMSE(\hat{\beta}_0)$	$\hat{\beta}_1$	$SE(\hat{\beta}_1)$	$RMSE(\hat{\beta}_1)$
1,000	BFGS Exponential	1.238	0.061	0.238	3.468	0.009	0.032
	BFGS Weibull	1.312	0.072	0.312	3.467	0.011	0.033
	BFGS MF Exponential	1.007	0.067	0.054	3.499	0.010	0.007
	BFGS MF Weibull	1.008	0.067	0.055	3.499	0.010	0.007
1,500	BFGS Exponential	1.245	0.049	0.245	3.466	0.008	0.034
	BFGS Weibull	1.323	0.059	0.323	3.464	0.009	0.036
	BFGS MF Exponential	1.007	0.054	0.047	3.499	0.008	0.007
	BFGS MF Weibull	1.007	0.054	0.048	3.499	0.008	0.007
2,000	BFGS Exponential	1.225	0.043	0.225	3.469	0.007	0.031
	BFGS Weibull	1.290	0.050	0.290	3.468	0.008	0.032
	BFGS MF Exponential	1.007	0.047	0.037	3.499	0.007	0.006
	BFGS MF Weibull	1.007	0.047	0.037	3.499	0.007	0.006

Note: True parameter values are $\beta_0 = 1$ and $\beta_1 = 3.5$.

Table A.6: Maximum Likelihood γ-Estimates for Experiment 8

#Obs.	Model	$\hat{\gamma}_0$	$SE(\hat{\gamma}_0)$	$RMSE(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$SE(\hat{\gamma}_1)$	$RMSE(\hat{\gamma}_1)$	$\hat{\gamma}_2$	$SE(\hat{\gamma}_2)$	$RMSE(\hat{\gamma}_2)$
	Experiment 8: MF Weibull D.G.P.									
1,000	BFGS MF Exponential	-1.547	1.217	0.950	2.129	0.490	0.387	3.396	0.708	0.594
	BFGS MF Weibull	-1.543	1.216	0.952	2.124	0.489	0.392	3.388	0.706	0.604
1,500	BFGS MF Exponential	-1.457	0.985	0.852	2.068	0.387	0.320	3.435	0.584	0.570
	BFGS MF Weibull	-1.464	0.980	0.848	2.055	0.384	0.323	3.403	0.578	0.571
2,000	BFGS MF Exponential	-1.584	0.918	0.767	2.076	0.365	0.283	3.375	0.521	0.468
	BFGS MF Weibull	-1.582	0.917	0.765	2.073	0.365	0.282	3.370	0.520	0.465

Note: True parameter values are $\gamma_0 = -2$, $\gamma_1 = 2$, and $\gamma_2 = 3$.

values reported for the BFGS MF Weibull and BFGS MF exponential models in the lower half of A.5 (and in Figure A.8) are comparable, and the corresponding SEs and RMSEs for these two BFGS MF models are effectively equivalent. The same can be said for the $\hat{\gamma}$'s recovered by the BFGS MF exponential and BFGS MF Weibull models in Experiment 8, which are reported in Table A.6. In this case, the BFGS MF exponential exhibits slightly lower bias than the BFGS MF Weibull when $N = 1,000$, but slightly higher bias than the BFGS MF Weibull when $N = 1,500$ or $N = 2,000$. Returning to Table A.5 and Figure A.8, we can also note that our (BFGS) MF survival models substantially outperform the (BFGS) non-MF survival models when the true d.g.p. includes misclassified failure cases. However, we further find in this regard that our BFGS MF models' estimates of uncertainty, and to a lessor extent RMSEs, in Tables A.5-A.6 are generally inferior to those obtained for the Bayesian MF models in Tables A.3-A.4. As was the case for Experiments 1-4, these latter patterns suggest that Bayesian MF models are preferable to BFGS MF models for applied research.

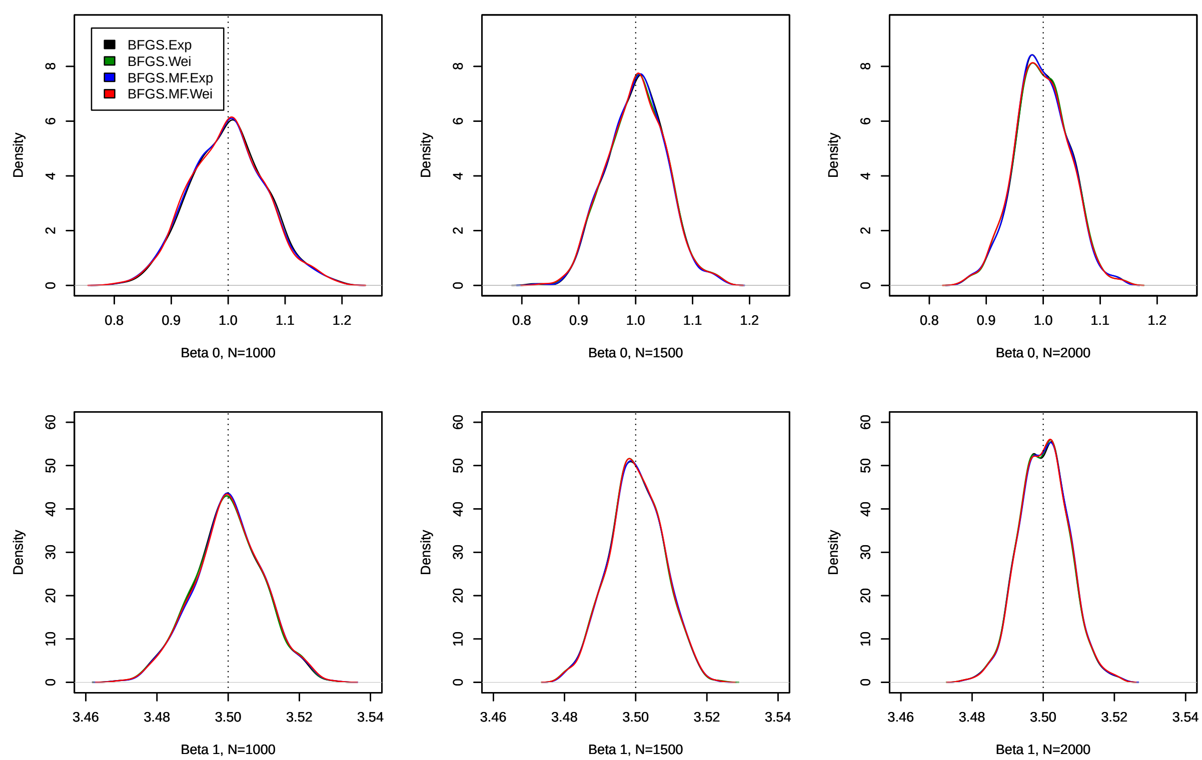


Figure A.7: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 7

Experiments 9-11 reevaluate the performance of the Bayesian Weibull and Bayesian MF

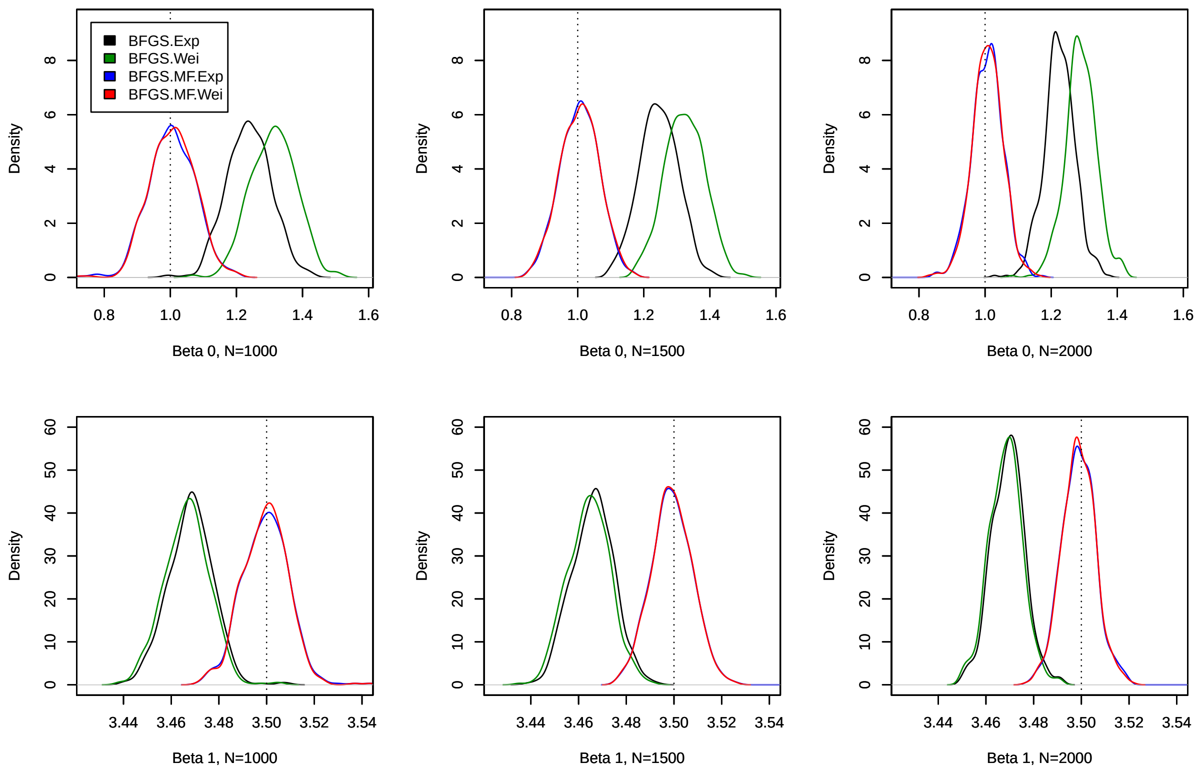


Figure A.8: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 8

Weibull models when applied to a MF Weibull-distributed outcome variable that exhibits a noticeably higher MF rate (i.e., α) than was the case for Experiments 2, 4, 6, and 8. More specifically, Experiments 9, 10, and 11 compare the relative performance of the Bayesian Weibull and Bayesian MF Weibull models when one's MF rate increases (from 5%) to 8%, 12%, and 15%, respectively. We report these MC results in Table A.7 (β parameters) and in Table A.8 (γ parameters). We also provide the full distributions of each estimated β parameter (i.e., across all 500 simulations) within Figures A.9-A.11. These tables and figures together demonstrate that the previously identified advantages of the Bayesian MF Weibull model (i.e., over the Bayesian Weibull model) become even more notable as one increases a Weibull-distributed outcome variable's MF rate from 5% to 8-15%. Beginning with Table A.7, we can note for example that the standard Bayesian Weibull model's mean $\hat{\beta}_0$'s substantially (and increasingly) overestimate β_0 as one's MF rate increases from 8%-to-15%; whereas the standard Bayesian Weibull model's $\hat{\beta}_1$'s increasingly underestimate β_1 under these same conditions.

Table A.7: Markov Chain Monte Carlo (MCMC) β-Estimates for Experiments 9-11

	Experiment 9: MF Weibull D.G.P. with $\alpha = 8\%$						
#Obs.	Model	$\hat{\beta}_0$	*MCSE*($\hat{\beta}_0$)	*RMSE*($\hat{\beta}_0$)	$\hat{\beta}_1$	*MCSE*($\hat{\beta}_1$)	*RMSE*($\hat{\beta}_1$)
1,000	Bayes Weibull	1.371	0.005	0.371	3.4620	0.001	0.038
	Bayes MF Weibull	1.000	0.003	0.026	3.500	0.000	0.004
1,500	Bayes Weibull	1.361	0.004	0.361	3.463	0.001	0.037
	Bayes MF Weibull	1.000	0.003	0.023	3.500	0.000	0.003
2,000	Bayes Weibull	1.376	0.003	0.376	3.461	0.000	0.039
	Bayes MF Weibull	1.000	0.002	0.018	3.500	0.000	0.003
	Experiment 10: MF Weibull D.G.P. with $\alpha = 12\%$						
#Obs.	Model	$\hat{\beta}_0$	*MCSE*($\hat{\beta}_0$)	*RMSE*($\hat{\beta}_0$)	$\hat{\beta}_1$	*MCSE*($\hat{\beta}_1$)	*RMSE*($\hat{\beta}_1$)
1,000	Bayes Weibull	1.489	0.004	0.489	3.446	0.001	0.054
	Bayes MF Weibull	1.001	0.003	0.027	3.500	0.000	0.004
1,500	Bayes Weibull	1.473	0.003	0.473	3.447	0.001	0.053
	Bayes MF Weibull	1.005	0.003	0.022	3.499	0.000	0.003
2,000	Bayes Weibull	1.462	0.003	0.462	3.499	0.000	0.051
	Bayes MF Weibull	1.005	0.002	0.020	3.499	0.000	0.003
	Experiment 11: MF Weibull D.G.P. with $\alpha = 15\%$						
#Obs.	Model	$\hat{\beta}_0$	*MCSE*($\hat{\beta}_0$)	*RMSE*($\hat{\beta}_0$)	$\hat{\beta}_1$	*MCSE*($\hat{\beta}_1$)	*RMSE*($\hat{\beta}_1$)
1,000	Bayes Weibull	1.559	0.004	0.559	3.435	0.001	0.065
	Bayes MF Weibull	1.011	0.004	0.030	3.499	0.001	0.004
1,500	Bayes Weibull	1.552	0.004	0.552	3.437	0.001	0.063
	Bayes MF Weibull	1.010	0.003	0.023	3.499	0.000	0.003
2,000	Bayes Weibull	1.557	0.003	0.557	3.436	0.000	0.064
	Bayes MF Weibull	1.009	0.002	0.020	3.499	0.000	0.003

Note: True parameter values are $\beta_0 = 1$ and $\beta_1 = 3.5$.

By contrast, the Bayesian MF Weibull model's averaged β estimates do not exhibit any notable trends in increasing (or decreasing) size as one increases the MF rate beyond 5%, suggesting that whereas Bayesian Weibull model's estimates become more biased as the MF rate increases, the Bayesian MF Weibull model remains comparatively unbiased. This contention is reinforced by the reported RMSEs in Table A.7, and Figures A.9-A.11. With regards to the former quantities, for example, we find in Table A.7 that our Bayesian MF Weibull model's $\hat{\beta}_0$'s exhibit RMSEs that are generally 16 times smaller than those of the Bayesian Weibull model when α=8%, and RMSEs that are generally 24 times smaller than those of the Bayesian Weibull model when α=15%. The findings for $\hat{\beta}_1$ are similar, and demonstrate that one's Bayesian MF Weibull exhibits RMSEs that are 12 times smaller than those of the Bayesian Weibull model when α=8%, and RMSEs that are generally 21 times smaller than those of the Bayesian Weibull model when α=15%. Turning next to the MF Weibull γ estimates for Experiments 9-11 (Table

Table A.8: Markov Chain Monte Carlo (MCMC) γ-Estimates for Experiments 9-11

#Obs.	Model	$\hat{\gamma}_0$	$MCSE(\hat{\gamma}_0)$	$RMSE(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$MCSE(\hat{\gamma}_1)$	$RMSE(\hat{\gamma}_1)$	$\hat{\gamma}_2$	$MCSE(\hat{\gamma}_2)$	$RMSE(\hat{\gamma}_2)$
	Experiment 9: MF Weibull D.G.P. with $\alpha = 8\%$									
1,000	Bayes MF Weibull	2.003	0.278	0.313	1.084	0.086	0.272	4.208	0.159	0.378
1,500	Bayes MF Weibull	1.962	0.192	0.407	1.077	0.049	0.192	4.167	0.116	0.442
2,000	Bayes MF Weibull	2.139	0.206	0.372	1.030	0.057	0.182	4.169	0.097	0.387
	Experiment 10: MF Weibull D.G.P. with $\alpha = 12\%$									
#Obs.	Model	$\hat{\gamma}_0$	$MCSE(\hat{\gamma}_0)$	$RMSE(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$MCSE(\hat{\gamma}_1)$	$RMSE(\hat{\gamma}_1)$	$\hat{\gamma}_2$	$MCSE(\hat{\gamma}_2)$	$RMSE(\hat{\gamma}_2)$
1,000	Bayes MF Weibull	-2.886	0.418	0.471	1.920	0.176	0.447	5.506	0.218	0.294
1,500	Bayes MF Weibull	-2.363	0.270	0.589	1.800	0.121	0.375	5.730	0.175	0.586
2,000	Bayes MF Weibull	-2.441	0.261	0.636	1.842	0.118	0.331	5.776	0.161	0.667
	Experiment 11: MF Weibull D.G.P. with $\alpha = 15\%$									
#Obs.	Model	$\hat{\gamma}_0$	$MCSE(\hat{\gamma}_0)$	$RMSE(\hat{\gamma}_0)$	$\hat{\gamma}_1$	$MCSE(\hat{\gamma}_1)$	$RMSE(\hat{\gamma}_1)$	$\hat{\gamma}_2$	$MCSE(\hat{\gamma}_2)$	$RMSE(\hat{\gamma}_2)$
1,000	Bayes MF Weibull	5.148	0.734	0.739	-1.028	0.232	0.475	6.424	0.720	0.424
1,500	Bayes MF Weibull	5.568	0.602	0.775	-1.179	0.135	0.478	5.977	0.502	0.677
2,000	Bayes MF Weibull	5.759	0.518	0.806	-1.013	0.094	0.343	5.985	0.278	0.685

Note: True parameter values are $\gamma_0 = 2$, $\gamma_1 = 1$, & $\gamma_2 = 4$ (Experiment 9); $\gamma_0 = -3$, $\gamma_1 = 2$, & $\gamma_2 = 5$ (Experiment 10); and $\gamma_0 = 4.5$, $\gamma_1 = -1$, & $\gamma_2 = 5$ (Experiment 11).

A.8), we can note that our averaged $\hat{\gamma}$ values for these Bayesian MF Weibull models generally recover one's true γ values at comparable rates to those of the Bayesian MF Weibull models in Experiments 2, 4, 6, and 8. However, we can again note that the Bayesian MF Weibull model's $\hat{\gamma}$'s in Table A.8 exhibit higher bias, and lower efficiency, than was the case for these same Bayesian MF Weibulls' $\hat{\beta}$'s in Table A.7.

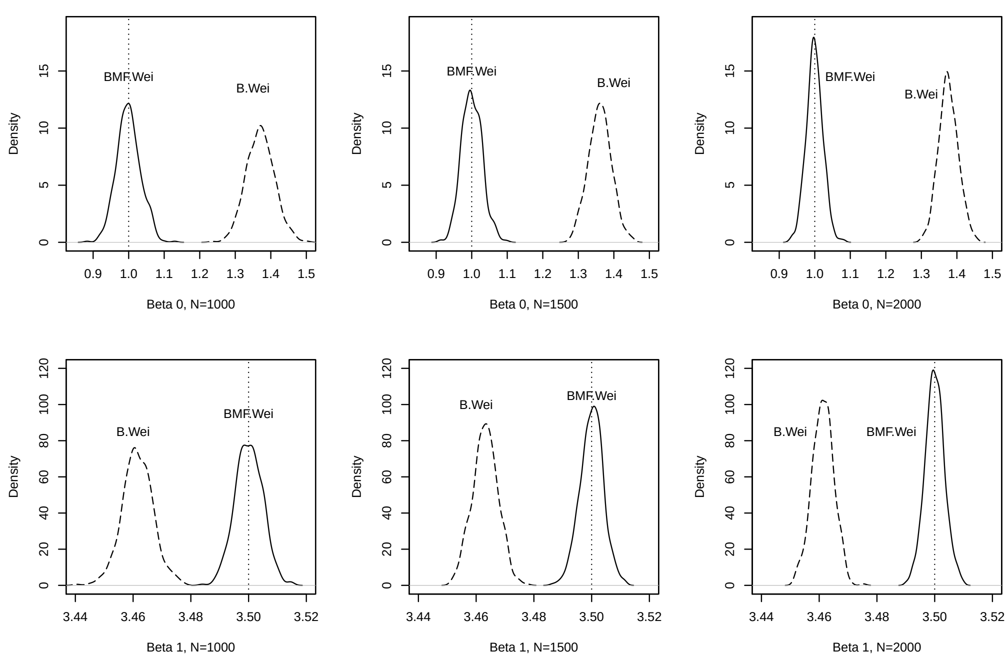


Figure A.9: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 9

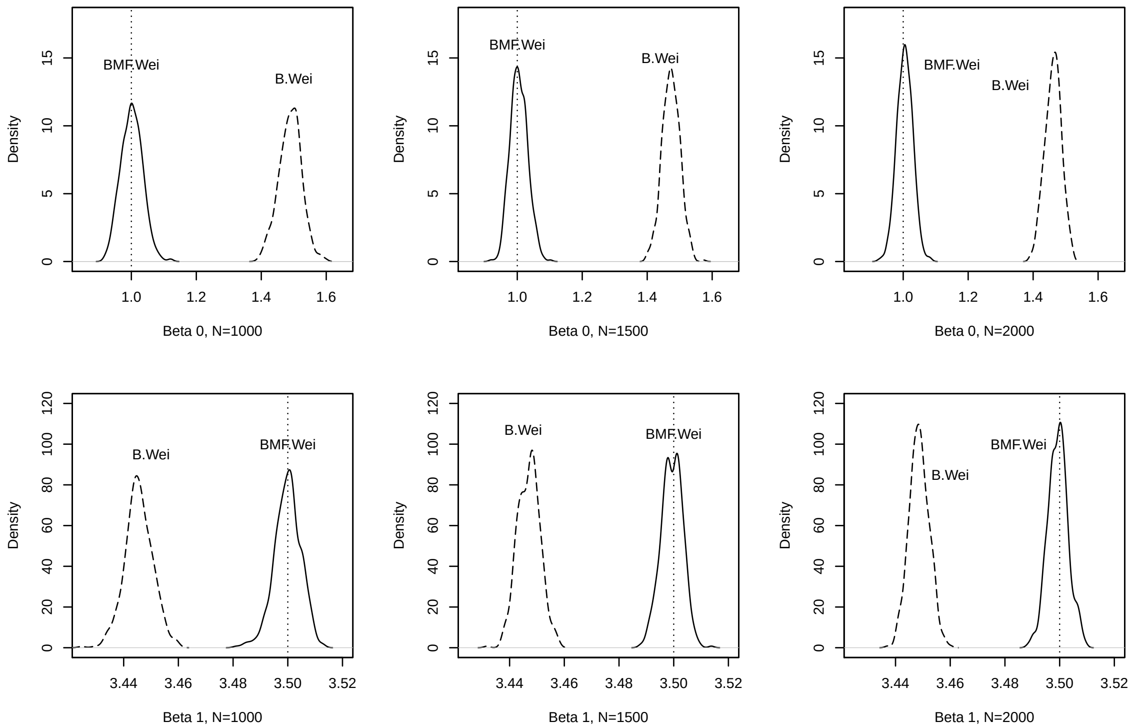


Figure A.10: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 10

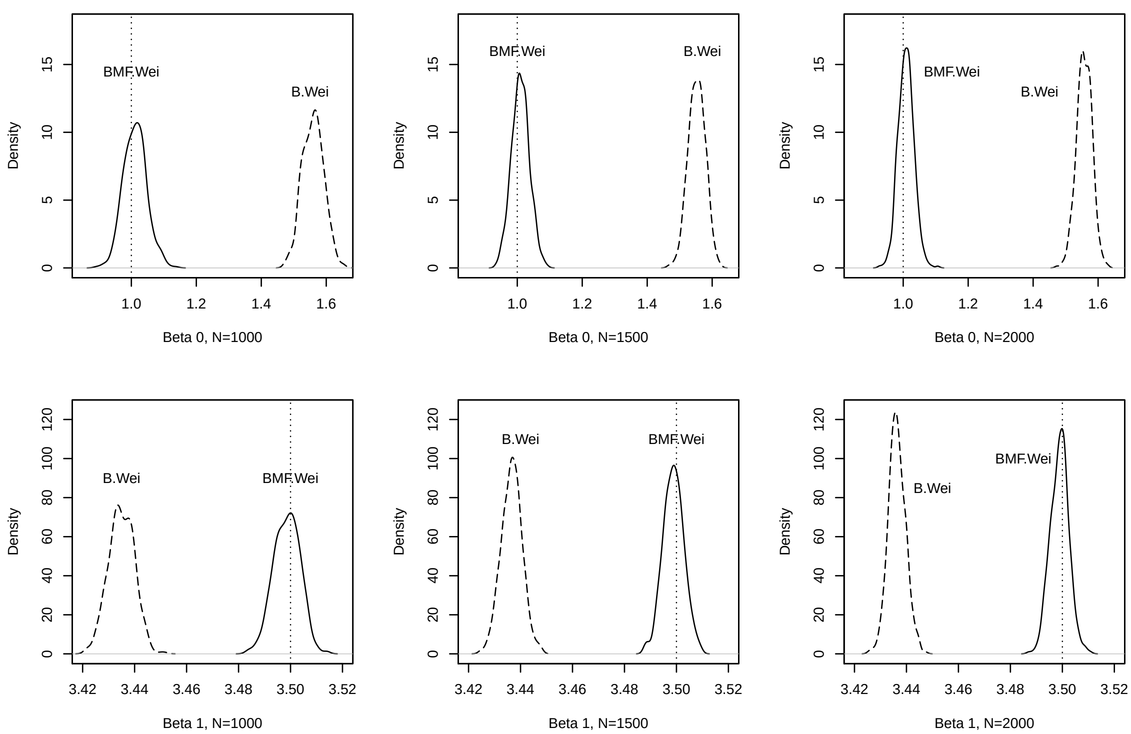


Figure A.11: Distributions of $\hat{\beta}$'s Across 500 Simulations for Experiment 11

IV Deviance Information Criterion

In order to compare the model fit of various models estimated in the empirical analyses, we calculated the Deviance Information Criterion (DIC) of our Bayesian (MF) Weibull models using the algorithm introduced in this section. First, the formula of DIC is

$$DIC = -2 * (L - P), \tag{A.20}$$

where L is the log likelihood of the data given the posterior means of the parameters ($\hat{\theta}$):

$$L = logp(\beta, \gamma, \lambda|\hat{\theta}), \tag{A.21}$$

and P is an estimate of the effective number of parameters in the model:

$$P = 2 * \Big[L - \frac{1}{S}\sum_{s=1}^{S} logp(\beta, \gamma, \lambda|\theta_s)\Big]. \tag{A.22}$$

In equation (A.22), S is the number of posterior samples and θ_s is the parameter vector for the s^{th} sample.

Based on the information above, the steps of calculating the DIC are as follows:

- **Step 1.** Obtain $\hat{\theta}$, a vector of posterior means of the parameters.
- **Step 2.** Calculate the log-likelihood of the data given $\hat{\theta}$ (i.e., L) using the respective Bayesian (MF) Weibull log-likelihood function(s).
- **Step 3.** Compute the log-likelihood of the data given the first posterior sample, θ_1. Repeat S times (for θ_2, θ_3, θ_4, ..., θ_S).
- **Step 4.** Obtain $\sum_{s=1}^{S} logp(\beta, \gamma, \lambda|\theta_s)$ by summing up the results from Step 3.
- **Step 5.** Calculate P using equation (A.22).
- **Step 6.** Calculate DIC using equation (A.20).

We calculate the DIC for each estimated Bayesian non-MF and MF Weibull model reported in Table 3 using Steps 1-6. A comparison of these DIC values show that in terms of overall fit, each Bayesian MF Weibull model in Table 3 is superior (with respect to DIC performance) compared to the Bayesian non-MF Weibull model reported in the table. Hence, while there is some variation in DIC peformance across the Bayesian MF Weibull models, the obtained DIC values show unambiguously that the estimated Bayesian MF Weibull models do better than the Bayesian non-MF Weibull model in terms of model fit for the Buhaug *et al* application.

V Civil War Application: Buhaug *et al* (2009)

We present below the results from additional tables and figures that were discussed in the main paper for the Buhaug *et al* (2009) application but which were not presented because of space constraints. To this end, first recall that we mentioned in the text that there are numerous Misclassified civil war "failure" cases in the Buhaug *et al* (2009) data. Examples of these misclassified failure cases in which the civil war is coded as "terminated" in the Buhaug *et al* (2009) data but which persisted beyond their terminated date are listed in table A.9.

Table A.9: Civil War Examples that Persisted Beyond "Failed" Date in Sample

Country	Civil War Case and Rebel Group(s)	Country	Civil War Case and Rebel Group(s)
Chad	Mouvement pour Démocratie et Développement-Forces Armées Occidentales (MDD-FAO)	Liberia	National Patriotic Front of Liberia (NPFL)
Philippines	Abu Sayyaf	Niger	Union des Fronts de la Résistance Armée (UFRA)
Philippines	Moro Islamic Liberation Front (MILF)	Peru	Sendero Luminoso
India	Ogaden National Liberation Front (ONLF)	Ethiopia	Tripura National Volunteers (TNV)
India	United Liberation Front of Assam (ULFA)	Egypt	El Gama'a El Islamiyya
India	Kuki National Front (KNF)	Pakistan	Muttahida Qaumi Movement (MQM)
Myanmar	Kachin Independence Organization (KIO)	Papua New Guinea	Bougainville Revolutionary Army (BRA)
Myanmar	Communist National Party (BCP)	DR Congo	Ninjas
Mozambique	RENAMO	Mali	Arab Islamic Front of Azawad (FIAA)
Sri Lanka	Liberation Tigers of Tamil Eelam (LTTE)	Indonesia	Freitlin: Revolutionary Front for an Independent East Timor

We also mentioned in the text that we derived posterior density plots of the key covariates from the misclassification stage (these are the $\boldsymbol{\gamma}$ covariates) and the survival stage (these are the $\boldsymbol{\beta}$ covariates) of the Bayesian MF Weibull specification in Model 4 (Table 3) that is estimated on the Buhaug *et al* (2009) data. We first present below the posterior density plots of the misclassification stage's $\boldsymbol{\gamma}$ covariates that essentially confirm the posterior mean estimate results of these covariates that are discussed in the text. This is followed by the presentation of the posterior density plots of the survival stage's $\boldsymbol{\beta}$ covariates, which also confirm the posterior mean estimate results for these covariates that are discussed in the text in the case of the Buhaug *et al* (2009) application.

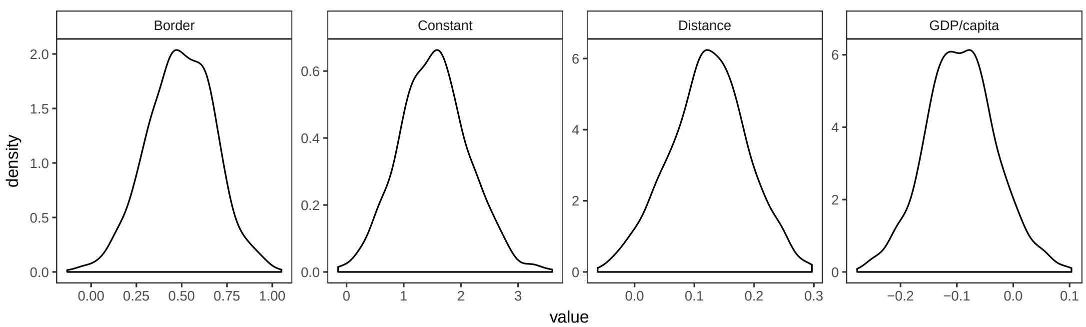


Figure A.12: Distribution of $\boldsymbol{\gamma}$ covariates for Model 4 (Table 3)

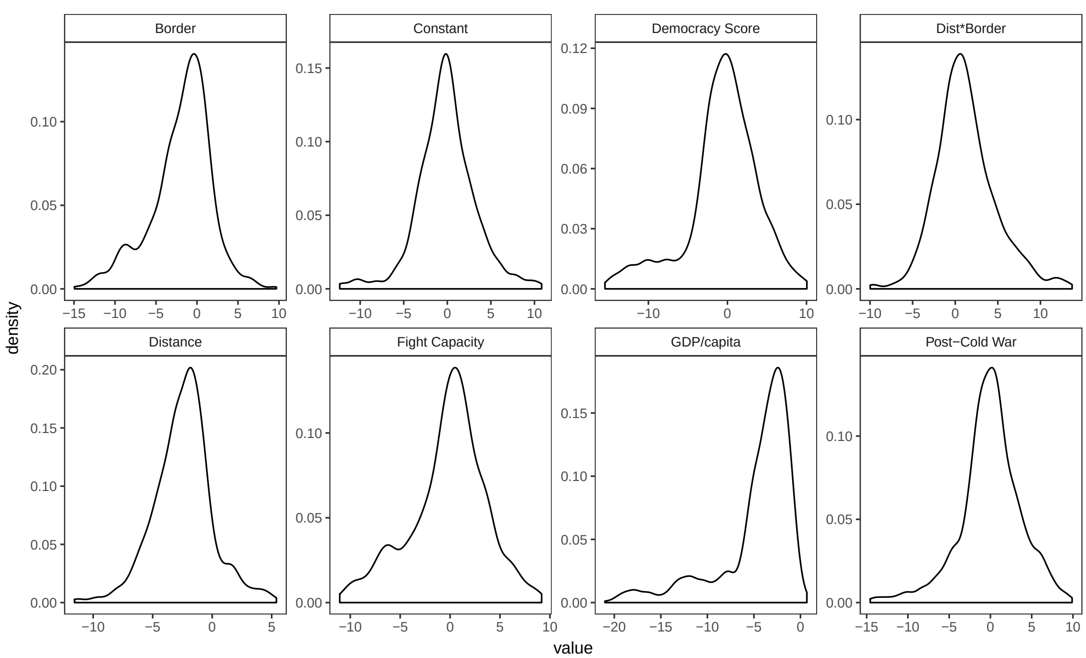


Figure A.13: Distribution of $\boldsymbol{\beta}$ covariates for Model 4 (Table 3)

Finally, we present below the results from several specification robustness tests for the Bayesian MF Weibull model estimated on the Buhaug *et al* (2009) application. The survival stage specification covariates in each Bayesian MF Weibull model reported in the following table are exactly the same as those reported in the Bayesian MF Weibull's survival stage in Table 3 in the text. We simply vary the number of covariates in the misclassification stage of each model reported below. For example, we include *Distance to capital (ln)* and *Conflict at border* in the misclassification stage of Model A below. In Model B, we add *GDP capita at onset*, *Democracy score at onset*, and *Post-Cold War years* to the misclassification stage specification. Finally, we add all covariates in the survival stage except for the *Distance* $\times$ *Border* and *Democracy score at onset* variables to the misclassification stage of Model C in Table A.10 below.

Table A.10: Buhaug *et al* (2009) Bayesian MF Model Robustness Test

	Bayesian MF Weibull		
	Model A	Model B	Model C
X *Covariates* (**β** *Parameters*)			
Distance to capital (ln)	-5.777 (-21.551, 3.943)	-3.254 (-12.131, 3.345)	-2.615 (-8.93, 3.193)
Conflict at border	-1.774 (-9.712, 5.737)	0.118 (-9.043, 12.571)	-0.79 (-9.601, 8.722)
Border×Distance	0.881 (-7.594, 10.791)	1.422 (-7.244, 13.777)	0.508 (-9.718, 11.321)
Rebel fighting capacity	1.819 (-9.61, 20.215)	-0.695 (-9.216, 10.369)	1.912 (-10.074, 15.892)
Democracy score at onset	0.824 (-7.753, 12.177)	-1.372 (-22.685, 8.312)	-0.681 (-12.997, 14.052)
GDP capita at onset (ln)	-4.32 (-11.976, -0.484)	-5.219 (-16.615, -0.36)	-7.358 (-17.802, -0.909)
Post-Cold War years	-0.341 (-11.963, 9.072)	-1.261 (-14.159, 8.169)	-1.31 (-16.13, 11.032)
Constant	-1.215 (-8.147, 5.826)	-0.172 (-9.842, 10.739)	1.061 (-7.406, 14.684)
Z *Covariates* (**γ** *Parameters*)			
Distance to capital (ln)	0.12 (-0.009, 0.254)		0.084 (-0.049, 0.217)
Conflict at border	0.482 (0.123, 0.805)		0.485 (0.115, 0.867)
Border× distance			
Rebel fighting capacity			-0.522 (-0.881, -0.147)
Democracy score at onset		0.668 (0.231, 1.117)	
GDP capita at onset (ln)		-0.009 (-0.157, 0.133)	-0.006 (-0.164, 0.149)
Post-Cold War years		-0.777 (-1.168, -0.412)	-0.745 (-1.115, -0.385)
Constant	0.918 (0.152, 1.764)	2.152 (1.259, 3.132)	1.679 (0.406, 3.064)
DIC	-11742.9	-17746.99	-10917.45
log likelihood	631.498	688.687	649.564
Observations	1,375	1,375	1,375

Note: Posterior means are reported with 95% credible intervals in parentheses for the Bayesian MF Weibull models.

The misclassification and survival stage results for the covariates in each of the Bayesian MF Weibull specifications are effectively the same as the results for these Bayesian MF Weibull covariates in Table 3. For instance, the posterior mean estimate of *GDP capita at onset* (*ln*) is consistently negative and the 95% BCI of this covariate always excludes zero in the survival stage of the Bayesian MF Weibull model. The mean estimate of *Post-Cold War years* years in the survival stage is also (as reported in the main paper) negative, although the 95% BCI of this estimate always includes zero. All other covariate results in the Table above are, as mentioned earlier, similar to the results for these covariates in Table 3.

VI Democratic Survival Application: RBS (2007)

The second application of our Bayesian MF Weibull model focuses on an important study of democratic regime survival conducted by Reenock, Bernhard and Sobek (2007) (hereafter RBS) for the 1961-1995 temporal period. The main aim of this study is to explain how the deprivation of basic needs of civilians (specifically, food insecurity) impacts the survival of democratic regimes at various levels of economic development. RBS (2007) argue that the level of deprivation—conceptualized and operationalized "as the reciprocal of the average daily per-capita caloric consumption (the inverse of basic needs satisfaction)" (RBS 2007, 687)—is a clear indicator of regressive socio-economic distribution. Higher levels of deprivation threaten democratic regime survival and increase the prospects of a political breakdown (i.e., regime failure) when the annual level of economic development per capita within a given country is moderately high. This is because in democracies with at least moderate income levels—that is, democracies with per capita income of $2,300 and beyond—citizens have sufficient socio-economic capacity to mobilize against incumbents and credibly threaten regime survival when deprivation of their basic needs increases (RBS 2007, 687).

RBS evaluate this moderated effect by interacting their main explanatory variable, *basic needs deprivation*, with *GDP per capita* (*logged*) to see how this interaction impacts their outcome variable, *democratic survival* (that is, democratic regime duration) (RBS 2007, 691) using a standard MLE Weibull model. In addition to this interaction term and its individual components, RBS also include the variables *economic growth*, the dummy variable *Presidentialism* for presidential democracies, *effective number of parties*, *religious fractionalization*, *ethnic frac-*

tionalization, *past attempts at democracy*, and the level of *trade openness*. RBS (2007) find that *basic needs deprivation* $\times$ *GDP per capita* (*logged*) reliably increases the hazard of democratic regime failure in their MLE Weibull model, which supports their theoretical prediction. With respect to substantive effects, they find that increasing *basic needs deprivation* from 1 SD below to 1 SD above its mean in democracies reliably increases the hazard of democratic regime "breakdown" (or equivalently decreases democratic regime duration) when per capita income in their sample of democracies reaches $2,300 (RBS 2007, 692). They also find that *economic growth* and *trade openness* reliably decreases the hazard of democratic regime breakdown while higher *religious fractionalization* substantially and reliably increases the hazard of democratic "failure" (RBS 2007, 692).

Despite the substantive value of RBS's findings, the criteria used to code failure—that is, democratic regime breakdown—likely means that observed democratic failures within their data are contaminated with latent misclassified failure cases. To see why, first note that to be included in RBS' analysis of democratic regime (survival), a country must meet Dahl's (1976) threshold of "polyarchy" (that is, a political system that is both competitive and inclusive) and Linz and Stephan's (1996) criteria of "stateness". Countries that cease to meet any of these standards are thus coded as "failed" (i.e., given a score of one) and exit the dataset. However, the decision of what constitutes an important enough decline in inclusiveness, stateness, and competitiveness as to justify a democratic regime being coded as "failed" is *inherently subjective*. Such criteria are prone to misinterpretation or subjective judgments about the date of democratic regime breakdown, which thus inadvertently leads to the misclassification of observed event-failures. Indeed, owing to their subjective criteria for identifying democratic regime breakdown, we find for example that RBS code breakdown of democracy in Thailand in 1976 and Sri Lanka in 1983. Yet secondary sources such as the political regimes dataset in Polity IV and primary sources (listed in Table A.13) show that democracy (as per Dahl's polyarchy criteria used by RBS) persisted in (i) Thailand beyond 1971 and also in (ii) Sri Lanka well beyond 1983 (in fact, into the 1990s as well). These two examples are hardly unique. In fact, numerous additional examples of recorded democratic-failure years listed in Table A.11 have been misidentified in the RBS (2007) data, suggesting that their observed democratic-failures are indeed contaminated with misclassified failure cases.

Table A.11: Democratic Regimes Examples in RBS (2007) that Survived Beyond "Failed" Date

Country	Event Description
Ghana	Recorded as Failed in 1972 in RBS (2007) but democracy survived (as per "competitiveness" criteria) in Ghana beyond this date according to secondary sources[1] and primary sources.[2]
Madagascar	Recorded as Failed in 1971 in RBS (2007). Yet democracy survived (as per "competitiveness" criteria) in Madagascar beyond this date according to secondary sources[1] and primary sources.[2]
Nigeria	Recorded as Failed in 1966 in RBS (2007). However, democracy survived (as per "competitiveness" criteria) in Nigeria well beyond this date (till the early 1970s) as per secondary sources[1] and primary sources.[2]
Malaysia	Recorded as Failed in 1969 in RBS (2007). But secondary sources[1] and primary sources[2] indicate that Malaysia made a transition to authoritarian rule only from the mid-1970s onwards which implies that it survived as a democracy well beyond this recorded date in RBS.
Peru	Recorded as Failed in 1992 in RBS (2007). Yet primary sources[2] have shown unambiguously that in terms of "inclusiveness" and "competitiveness," Peru continued as a democracy till the mid-1990s.
Uruguay	Recorded as Failed in 1973 in RBS (2007) but democracy survived (as per "inclusiveness" and "competitiveness" criteria) in Uruguay beyond this date according to secondary sources[1] and primary sources.[2]
Philippines	Recorded as Failed in 1972 in RBS (2007) but democracy survived (as per "inclusiveness" and "competitiveness" criteria) in Philippines beyond this date according to secondary sources[1] and primary sources.[2]
Suriname	Recorded as Failed in 1989 in RBS (2007). Yet democracy survived (as per "inclusiveness" and "competitiveness" criteria) in Madagascar beyond this date as according to secondary sources[1] and primary sources.[2]

Notes: [1]These secondary sources include the Varieties of Democracy (V-Dem) Database by Coppedge *et al* (2016), Cheibub *et al* (2010) political regimes dataset, and the Polity IV database. [2]For a list of these primary sources see Table A.13.

Given that contaminated misclassified failure cases in survival data generates econometric challenges, we estimate our Bayesian MF Weibull model (using the slice sampling MCMC algorithm) on RBS' survival data to statistically account for the possibility that the observed democratic regime failure in these data include misclassified failure cases. We replicate the main specification in column 2 of Table 1 in the RBS paper (see RBS 2007, 692 that tests the effect of *basic needs deprivation* $\times$ *GDP per capita* (logged) and other controls on their *democratic survival* outcome variable, first estimating (i) the standard MLE Weibull hazard model and then (ii) three different specifications of our Bayesian MF Weibull model on the RBS (2007) data using our slice sampling (MCMC) algorithm. To this end, we specified the Bayesian MF Weibull model's hyperparameters as follows: $a = 1$, $b = 1$, $S_\beta = I_{p1}$, $S_\gamma = I_{p2}$, $\nu_\beta = p1$ and $\nu_\gamma = p2$. The results from the Bayesian MF Weibull models are based on a set of 100,000 iterations with 10,000 burn-in scans and a thinning of 100.

To begin with, the standard MLE Weibull model in Model D in Table A.12 includes all the same variables reported in column 2 of Table 1 in RBS (2007) which influence the *democratic survival* variable (e.g., *basic needs deprivation* $\times$ *GDP per capita* (*logged*), the individual components of this interaction term, *economic growth*, *Presidentialism*, and so on). Next, in Model E of Table A.12, we report a baseline Bayesian MF Weibull specification in which the survival ($\mathbf{X}$) stage of this specification also includes all the same variables reported in column 2 (Table 1) in RBS (2007, 691). The misclassification stage of this baseline specification only includes an intercept.

The survival stage of the Bayesian MF Weibull specification in Model F (Table A.12) also includes all of the covariates reported in RBS (2007, 691). But the misclassification ($\mathbf{Z}$) stage in Model F includes the following covariates that may affect the likelihood that some cases of democratic breakdown have been misclassified as terminated even when they had (possibly) not failed. We first include the dummy variable for Presidential democracies (labeled *Presidentialism*) in the misclassification stage. To understand why, note that extensive debates exist about whether Presidential regimes are "inclusive" and "competitive" and the extent to which these regimes are inclusive and competitive [3] – two key criteria in Dahl's polyarchy concept that RBS use to code when democratic regimes breakdown in their sample. Because the extent of inclusiveness and competitiveness in Presidential regimes are ambiguous, it may be difficult for researchers to accurately identify if and when breakdown of Presidential democracies occur using the criteria that RBS (2007) employ. This increases the possibility of misidentification

[3] For this see e.g., Linz (1994); Samuel and Shugart (2010).

of breakdown of Presidential democracies. Hence, we anticipate that *Presidentialism* will be positively associated with the probability of misclassified failure.

Next, we add *GDP per capita* (logged) and *economic growth* to the misclassification stage in Model F. The rationale for doing so is as follows. Specifically, studies have shown that the frequency of democratic breakdown is rare in democracies with a relatively high per capita income level of $6,055 (1985 PPP USD) and beyond,[4] during periods of economic growth in democracies and in states characterized by high levels of trade openness (Przeworski and Limongi 1997; Boix 2003). Misclassifying or misidentifying democratic breakdowns under the aforementioned conditions is therefore less likely given the relative low frequency of democratic regime failure in the context of these conditions. Thus, we anticipate that logged *GDP per capita*, *economic growth*, and *trade openness* will each be negatively associated with the probability of misclassified failure. Moreover, unlike presidential democracies, scholars have suggested that the extent of formally institutionalized inclusiveness in democracies with higher levels of ethnic or religious fractionalization is well defined and clear (Munck 2009; Teorell 2010). It is thus easier to accurately record if and when democratic regime breakdown occurs in more fractionalized societies as per the criteria used by RBS (2007). Hence the influence of *ethnic* and *religious fractionalization* on the probability of misclassified failure is likely to be negative. Finally, Model G in Table A.12 includes all the covariates from the RBS (2007) specification within both the survival and misclassification stage of the Bayesian MF Weibull specification. We turn to first discuss the misclassification stage and then the survival stage results from the Bayesian MF Weibull models that are applied to the RBS (2007) democratic survival data.

The posterior density plots (see Figure A.14), posterior mean estimates for *ethnic* and *religious fractionalization* and the 95% BCI of these mean estimates in the misclassification stage reported in the bottom rows of Models F and G in Table A.12 statistically supports our claim that each of these two covariates will be negatively associated with the probability of misclassified failure. More specifically, the first difference in misclassification probabilities derived from Model F, which is illustrated in Figure A.15, shows that increasing *ethnic* and

[4] Przeworski and Limongi 1997, 165.

Table A.12: RBS (2007) (Bayesian MF) Weibull Model Results

	MLE Weibull	Bayesian MF Weibull		
	Model D	Model E	Model F	Model G
X *Covariates* (*$\boldsymbol{\beta}$ Parameters*)				
Basic needs deprivation (1/calorie consumption)	-65889.29 (20586.12)	-0.15 (-3.859, 3.669)	-0.1 (-6.262, 5.684)	-0.499 (-16.883, 11.713)
GDP per capita (logged)	-5.586 (1.387)	-0.96 (-1.276, -0.643)	-1.33 (-7.175, 3.459)	-2.072 (-15.198, 5.5)
Basic needs deprivation× GDP per capita (logged)	9469.909 (2926.242)	-0.247 (-3.645, 4.564)	-0.531 (-9.293, 5.815)	-0.621 (-9.173, 6.728)
Economic growth	-6.87 (3.496)	-2.119 (-5.832, 0.863)	-0.514 (-10.556, 6.565)	1.06 (-15.973, 16.192)
Presidentialism	0.335 (0.398)	0.283 (-0.263, 0.827)	0.52 (-4.327, 7.162)	-0.571 (-11.584, 7.617)
Effective no. parties	0.028 (0.159)	0.041 (-0.193, 0.237)	-1.405 (-12.93, 4.984)	-0.247 (-8.171, 7.647)
Religious fractionalization	2.683 (1.132)	1.447 (0.117, 2.806)	-0.402 (-8.912, 5.749)	0.969 (-6.387, 9.332)
Ethnic fractionalization	-0.431 (1.296)	1.189 (0.001, 2.46)	-0.489 (-9.792, 6.001)	0.035 (-9.337, 11.457)
Past attempts at democracy	-0.03 (0.533)	0.042 (-0.579, 0.614)	0.16 (-4.493, 5.731)	3.084 (-4.178, 16.813)
Trade Openness (current dollars)	-0.021 (0.009)	-0.011 (-0.024, -0.001)	-2.86 (-8.091, -0.012)	-3.904 (-11.524, -0.193)
Constant	35.167 (11.101)	2.574 (0.138, 5.614)	-0.261 (-8.182, 5.86)	0.408 (-7.596, 9.468)
Z *Covariates* (*$\boldsymbol{\gamma}$ Parameters*)				
Basic needs deprivation (1/calorie consumption)				0.084 (-4.968, 6.577)
GDP per capital (logged)			1.301 (0.696, 4.508)	1.098 (0.64, 1.647)
Basic needs deprivation× GDP per capita (logged)				-0.341 (-6.233, 5.141)
Economic growth			3.807 (-0.887, 9.523)	3.316 (-0.787, 8.385)
Presidentialism			-0.248 (-1.226, 4.288)	-0.459 (-1.194, 0.186)
Effective no. parties				-0.011 (-0.245, 0.296)
Religious fractionalization			-1.697 (-3.522, -0.092)	-1.725 (-3.267, -0.298)
Ethnic fractionalization			-0.673 (-2.475, 1.088)	-0.782 (-2.401, 0.924)
Past attempts at democracy				0.257 (-0.462, 1.035)
Trade openness (current dollars)			0.09 (0.006, 0.364)	0.02 (0.007, 0.034)
Constant		27.172 (5.554, 70.924)	-4.57 (-9.689, -0.188)	-4.513 (-9.271, -0.541)
DIC		-8906.94	-5693.795	-2326.107
AIC	158.402			
log likelihood	-67.201	264.747	155.642	339.687
Observations	1722	1722	1722	1722

Notew: Posterior means are reported with 95% credible intervals in parentheses for the Bayesian MF Weibull models. Variable coefficients are reported with standard errors clustered by country in parentheses for the MLE Weibull model. For the MLE Weibull model, we replicate RBS' results in hazards from as opposed to the accelerated time failure (AFT) form reported by RBS in the paper for better comparison with the Bayesian MF Weibull models.

religious fractionalization from 1 SD below to 1 SD above their respective mean[5] decreases the probability of misclassified failure by approximately (i) 5.8% for *religious fractionalization* and (ii) 3.2% in the case of *ethnic fractionalization.* The 95% BCI of the substantive effect of *religious fractionalization* excludes zero, while the 95% BCI of *ethnic fractionalization* includes zero. The former result confirms our intuition that it is less likely that regime failure in democracies with high levels of religious fractionalization will be misclassified, and moreover, this result is reliable.

Contrary to our expectations, we find from the misclassification stage of the Bayesian MF

[5]While holding other covariates at their respective mean or mode in the sample.

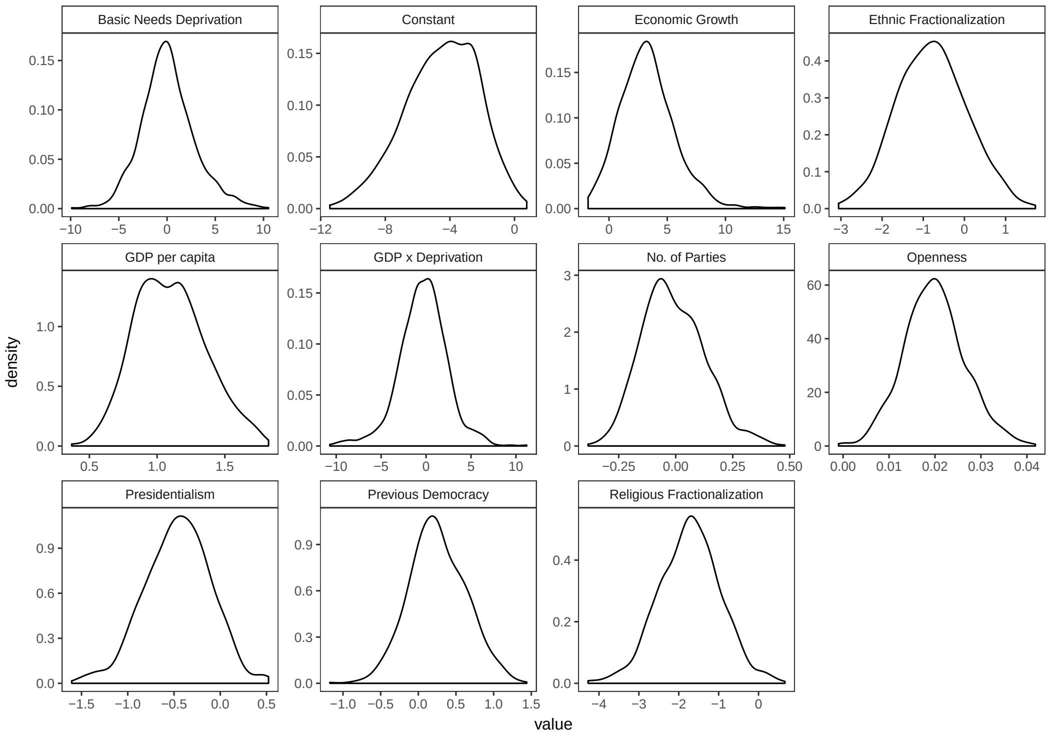
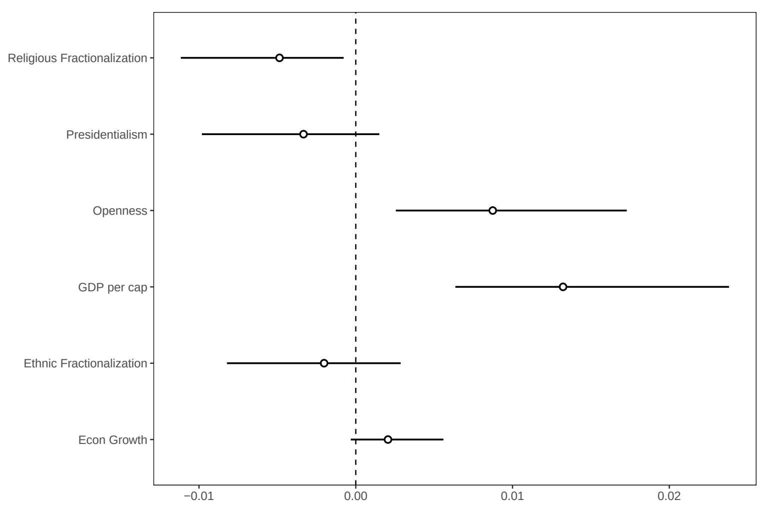


Figure A.14: Distribution of $\boldsymbol{\gamma}$ covariates for RBS MF Model G (Table A.12)



Note: Black dot represents predicted change in civil war duration as the row variable changes from 1 SD below the mean to 1 SD above the mean for continuous variables and 0-1 change for dichotomous variables while holding all other variables at their mean or mode. Whiskers indicate the 95% credible interval. Results come from Model 8 in Table A.12.

Figure A.15: Change in Probability of Misclassification for Reenock *et al.* (2007)

Weibull specifications that *GDP per capita* (logged), *economic growth* and *trade openness* are each positively associated with the probability of misclassified failure. The first difference in misclassification probabilities with 95% BCI illustrated in Figure A.15 shows that the statistically positive association between two of the three aforementioned covariates (*GDP per capita* (logged) and *trade openness*) and the probability of misclassified failure is indeed reliable and substantial. The posterior mean estimate for *Presidentialism* in the misclassification stages of the Bayesian MF Weibull models in A.12 is -0.248 and -0.459. But the 95% BCI of the mean estimate and substantive effect of *Presidentialism* (see figure A.15) in the misclassification stage always includes zero, which indicates that the association between this dummy variable and the probability of misclassified failure is unreliable.

With respect to the survival stage results, first note that the empirical insights from the MLE Weibull specification's estimates in Model D Table A.12 mirror those reported by RBS (2007: 689). The negative effect of *basic needs deprivation* $\times$ *GDP per capita* (logged) in the standard Weibull specification is reliable. It thus supports the finding by RBS (2007) that this interaction term reliably <u>increases</u> the hazard of democratic regime failure or equivalently decreases the duration of democratic regimes. The marginal effect of *basic needs deprivation* $\times$ *GDP per capita* (logged) that we derived from the standard MLE Weibull model reveals (as RBS find) that increasing deprivation of basic needs from 1 SD below to 1 SD above its mean reliably *decreases* the duration of democratic regimes by (ii) 11.3% when per capita income is held at the $2,300 threshold (the benchmark that RBS use); and (ii) 9.5% when income per capita is held at its mean of $6,443. In sharp contrast to this result by RBS (2007), the illustrated posterior density plots in Figure A.16 and posterior mean estimate of *basic needs deprivation* $\times$ *GDP per capita* (logged) in all the Bayesian MF Weibull survival stage specifications suggest that this interaction term <u>decreases</u> the hazard of democratic regime failure or, in other words, increases the duration of democracies even though the 95% BCI of this mean estimate consistently includes zero.

Importantly, the marginal effect of the *basic needs deprivation* $\times$ *GDP per capita (logged)* interaction term that we derive from the Bayesian MF Weibull survival stage specification in

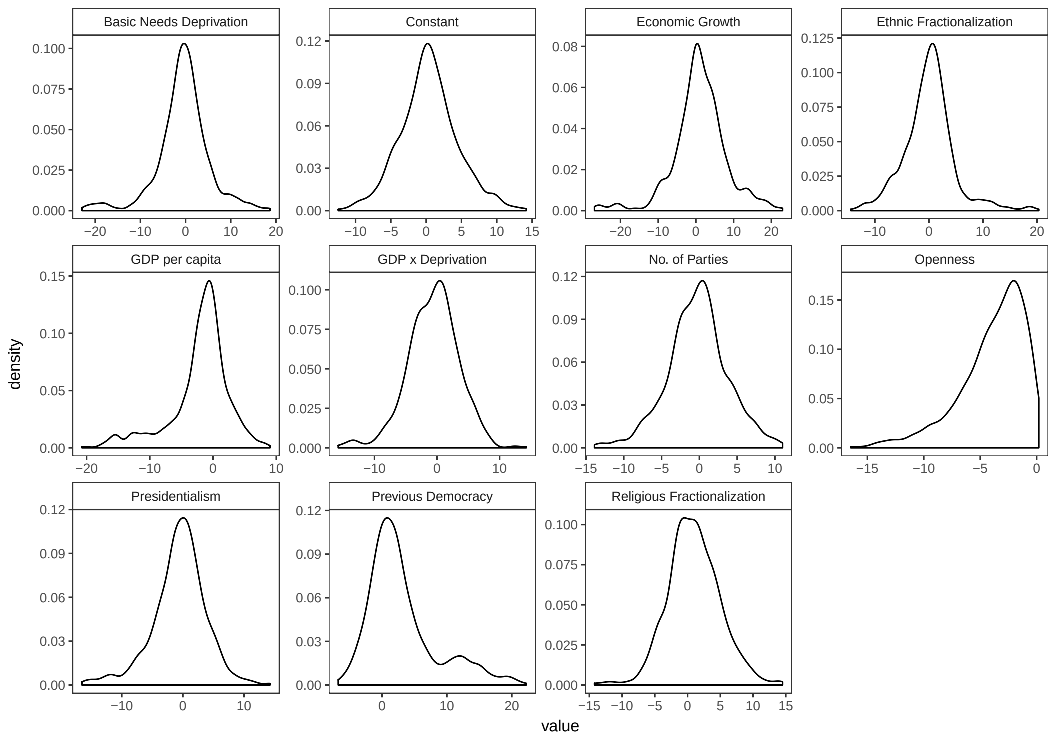


Figure A.16: Distribution of $\boldsymbol{\beta}$ covariates for RBS MF Model G (Table A.12)

Model F in A.12 reveals that increasing *basic needs deprivation* from 1 SD below to 1 SD above its mean *increases* the duration of democratic regimes by (i) 13.08% when per capita income is held at the $2,300 threshold (the RBS benchmark); and (ii) 24.5% when income per capita in is held at is mean of $6,443. The 95% BCI of these substantive effects excludes zero which thus indicates that it is reliable to infer that greater deprivation of basic needs actually increases (decreases) the duration (hazard) of democratic regimes (failure) when per capita income in democracies crosses the $2,300 threshold. This result is exactly the opposite of what RBS (2007) find in their standard MLE Weibull model. It also indicates that once we statistically account for misclassified democratic regime failure cases, we find that there is a negative–rather than positive (as RBS 2007 find)– association between basic needs deprivation and the hazard of democratic regime failure once per capita income reaches (and crosses) the moderately high $2,300 threshold.

Although the key interaction term's result in the Bayesian MF Weibull model's survival stage is substantially different from those reported by RBS (2007), the rationale underlying this "contrarian" result is intuitive and is as follows. Specifically, democracies with moderately high levels of per capita income and beyond are on average more accountable to their citizens and also have greater material capacity for addressing crises such as food (a key "basic need") shortages that result from exogenous shocks (e.g., Sen 1982; Lindert 2004). Hence when basic needs deprivation occur in relatively higher income democracies, governments in these democracies are more likely to successfully resolve such deprivation crises. This will serve to reinforce the citizens' faith in the democratic political process in these countries which in turn helps to increase the prospects for survival and duration of democracies, as shown empirically by our Bayesian MF Weibull models.

Other key results from the survival stage of all the Bayesian MF Weibull models in Table A.12 also vary dramatically from those reported by RBS (2007). For instance, RBS (2007) suggest and find that *economic growth* reliably decreases the hazard of democratic regime failure. The posterior density plot (see Figure A.16), posterior mean estimate and its 95% BCI for *economic growth* across the Bayesian MF Weibull's survival stage specifications, however, reveal that the association between this variable and the hazard of democratic regime failure is weak, inconsistent and fragile. RBS (2007) report that *religious fractionalization* reliably increases the hazard of democratic regime failure. In contrast, the posterior density plot, mean estimate and its 95% BCI for *religious fractionalization* in the full Bayesian MF Weibull model's survival stage specifications is also weak and inconsistent, therein suggesting that the association between *religious fractionalization* and the hazard of democratic regime failure is unreliable. Lastly, standard diagnostic checks for the parameters in each Bayesian MF Weibull specification estimated for the RBS (2007) data also suggests that the Markov chain has reached a steady state in each case. Indeed, trace plots (available on request) show that the Markov chain (i) has stabilized and appears constant over the graph, (ii) has good mixing and (iii) is dense (it also traverses the posterior space rapidly). The autocorrelation plots indicate no high degree of autocorrelation for the posterior samples, implying good mixing.[6] Thus, these plots indicate that the Markov chain has successfully converged to the desired posterior.

[6] *T*he "Geweke-Diagnostics" also show that the mean estimate of the Markov chain is stable over time.

Table A.13: Sources for Identifying Misclassified Democratic Failure Cases in RBS (2007) Data

Countries in sample by region	Region and country-specific sources
South and South-East Asia Bangladesh; Cambodia; India; Indonesia; Malaysia; Myanmar; Nepal; Pakistan; Philippines; Sri Lanka; Suriname; Thailand	Human Rights Watch. *Descent into Chaos: Thailand's 2010 Protests and the Government Crackdown.* New York and Bangkok: Human Rights Watch, 2011. Brass Paul (ed). *Routledge handbook of South Asian politics: India, Pakistan, Bangladesh, Sri Lanka, and Nepal.* Routledge, New York. Hussain . 2008. Politics of alliances in Pakistan, unpublished PhD thesis submitted to the National Institute of Pakistan Studies, Quaid-i-Azam University, Islamabad, Pakistan. Kochanek SA. 2000. "Governance, patronage politics, and democratic transition in Bangladesh." *Asian Survey* 40(3): 530550 Kershaw, Roger. 2001. *Monarchy in South-East Asia: The Faces of Tradition in Transition.* London and NewYork: Routledge. Case, William F. 2002. *Politics in Southeast Asia: Democracy or Less.* London and New York: Routledge Curzon. Connors, Michael K. 2011, "Ambivalent about Human Rights: Thai Democracy," in Thomas W. D. Davis and Brian Galligan (eds), *Human Rights in Asia*, Cheltenham, UK and Northampton, MA: Edward Elgar, pp. 10322 Freedman, Amy L. 2007, "Consolidation or Withering Away of Democracy? Political Changes in Thailand and Indonesia." *Asian Affairs: An American Review* vol. 33, no. 4 (Winter), pp. 195-216. Frolic, Michael B. 2001, "Transitions to Democracy after the Cold War," in Amitav Acharya, B. Michael Frolic, and Richard Stubbs (eds), *Democracy, Human Rights and Civil Society in South East Asia*, Toronto, Canada: University of TorontoYork University Joint Centre for Asia Pacific Studies, pp. 21–35.
Latin America Colombia; Ecuador; El Salvador; Dominican Republic; Guatemala; Grenada; Honduras; Mexico; Nicaragua; Paraguay; Peru; Suriname; Uruguay; Venezuela	Mettenheim, Kurt., and James Malloy. 1998. *Deepening Democracy in Latin America.* Pittsburgh: University of Pittsburgh Press. Millet, Richard. 2009. "Democratic Consolidation in Latin America?" In *Latin American Democracy : Emerging reality or endangered species?* eds, Richard L. Millet, Jennifer S. Holmes, Orlando J. Prez. New York: Routledge Pinkney, Robert. 2003. *Democracy in the Third World.* London: Lynne Rienner Publishers. Hagopian, Frances. and Scott Mainwaring., 2005. The Third Wave of *Democratization: Advances and Setbacks.* New York: Cambridge University Press Hagopian, Mainwaring, and Daniel Brinks. 2008. "Political Regimes in Latin America, 1900-2007" `http://kellogg.nd.edu/scottmainwaring/Political_Regimes.pdf` Rector, John. 2003. The History of Chile. New York: Palgrave Macmillan Schedler, Andreas. 2001. "Measuring Democratic Consolidation." Studies in *Comparative International Development* 36 (1): 66-92. Blake, Charles. 2005. *Politics in Latin America.* Boston: Houghton Mifflin Company. Buxton Julia., and Nicola Philips. 1999. *Case Studies in Latin America Political Economy.* Manchester: Manchester University Press.

Countries in sample by region	Region and country-specific sources
Africa Benin; Botswana; Chad; Congo; Gambia; Ghana; Madagascar; Mali; Mozambique; Namibia; Niger; Nigeria; Sierra Leone; South Africa; Sudan; Tanzania; Uganda; Zambia	Bogaards, M. 2004. "Counting parties and identifying dominant party system in Africa." *European Journal of Political Research.* 43: 173-197. Bratton, M., Walle, N. van de. 1997. *Democratic experiments in Africa.* Cambridge: Cambridge University Press. Osei, A. 2012. Party-Voter Linkage in Africa, Party Research in Africa: Findings and Problems. Ellis, S. 1994. "Democracy and Human Rights in Africa" in Rob Van Berg UlbeBosma (eds) *Poverty and Development: Historical Dimension of Development, Change and Conflict in the South*, Ministry of Foreign Affairs. The Hague. Pp. 115-124. Bratton, M., Houessou, R. 2014. *Demand for democracy is rising in Africa, but most political leaders fail to deliver.* Afrobarometer Policy Paper No. 11. Bratton, M., Mattes, R., Gyimah-Boadi, E. 2005. *Public opinion, democracy, and market reform in Africa.* New York: Cambridge University Press. Gyimah-Boadi, E. 2015. "Africa's waning democratic commitment." *Journal of Democracy*, 26(1), 101–113. LeBas, A. 2014. "The perils of power sharing." *Journal of Democracy.* 25(2), 52–66 Wing, S. 2008. *Constructing democracy in transitioning societies in Africa: Constitutionalism and deliberation in Mali.* New York: Palgrave Macmillan.
Central and Eastern Europe; Former USSR Austria; Czechoslovakia; Estonia; Germany; Hungary Latvia; Lithuania; Poland; Romania; Russia (USSR); Slovakia; Ukraine	Linz, Stepan, and Richard Gunther. 1995."Democratic Transitions and Consolidation in Southern Europe, with Reflections on Latin America and Eastern Europe." In *The Politics of Democratic Consolidation: Southern Europe in Comparative Perspective*, eds Richard Gunther, Nikiforos Diamandouros, Hans-Jurgen Puhel. Baltimore: John Hopkins University Press Plasser, Fritz and Ulram, Peter A. 1994. "Monitoring Democratic Consolidation: Political Trust and System Support in East-Central Europe", Paper for the XVI th World Congress of the International Political Science Association, Berlin. Banac, Ivo. 2014. "Twenty-Five Years after the Fall of the Berlin Wall. East European Politics and Societies and Cultures," *Special Issue on the Post-1989 Developments in the Region*, 28 (2): 653-657. Bogaards, Matthijs. 2009. "How to classify hybrid regimes? Defective democracy and electoral authoritarianism." *Democratization*, 16 (2): 399-423. Coman, Ramona and Tomini, Luca 2014. "A Comparative Perspective on the State of Democracy in Central and Eastern Europe." *Europe-Asia Studies* 66 (6): 853-858 Dzihic, Vedran. 2014. "Grey zones between democracy and authoritarianism: Re-thinking the current state of democracy in Eastern and South Eastern Europe" in Wiersma et al., eds. 2014. *Problems of Representative Democracy in Europe*, Amsterdam: Foundation for European Progressive Studies: 21-32. Papadopoulos, Yannis. 2013. *Democracy in Crisis? Politics, Governance and Policy.* Houndmills: Palgrave-Macmillan, 296.

Countries in sample by region	Region and country-specific sources
Central and Eastern Europe; Former USSR (Cont.) Austria; Czechoslovakia; Estonia; Germany; Hungary Latvia; Lithuania; Poland; Romania; Russia (USSR); Slovakia; Ukraine	Pappas, Takis. 2014. Populist Democracies: Post-Authoritarian Greece and Post-Communist Hungary. *Government and Opposition* 49 (1): 1-25. Golosov, G. 2008. "Electoral authoritarianism in Russia." *Pro et Contra*, JanuaryFebruary. Rose R, W. Mishler, N. Munro. 2006. *Russia transformed: developing popular support for a new regime.* Cambridge University Press, Cambridge, UK (2006) Rose R, N. Munro, W. Mishler. 2004. "Resigned acceptance of an incomplete democracy: Russia's political equilibrium." *Post-Soviet Affairs*, 20(3):195–218

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